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ACTIVE WALKS: THE FIRST TWELVE YEARS (PART II)

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Active Walk (AW) is a paradigm for self-organization and pattern formation in simple and complex systems, originated by Lam in 1992. In an AW, the walker changes the deformable landscape as it walks, and is influenced by the changed landscape in choosing its next step. Active walk models have been applied successfully to various biological, physical, geological and economic systems from both the natural and social sciences. More recently, it has been used to model human history. In Part I of this review, the birth of the AW paradigm, its basic concepts and formulations, a solvable two-site model, and the experiments and AW modeling of surface-reaction filamentary patterns are presented. Part II here continues with properties of AW, and applications of AW in nonliving and living systems — including those from the social sciences and human history. (In particular, unsuspected *quantitative* laws and a prediction about the Chinese history are given.) A comment on the relationship between physics, social science and complex systems is provided. The review concludes with open problems in the form of workable research projects and general discussions.

Keywords: Active walk; bacteria; ant; optimization; history; social science; complex system.

Contents

1. Introduction

Active Walk (AW) is a paradigm for self-organization and pattern formation in complex systems, originated by Lam in 1992 [Lam *et al.*, 1992; Freimuth & Lam, 1992]. In an AW, the walker (a particle with or without internal states) changes the deformable landscape as it walks and is influenced by the changed landscape in choosing its next step. AW may be viewed from three different perspectives, viz. as an organizing principle, as a new kind of walk in modeling, and as agents used in simulations [Lam, 2005a]. Active walk models have been applied successfully to various systems from both the natural and social sciences.

In Part I of this review [Lam, 2005a], the birth of the AW paradigm, its basic concepts and formulations, a solvable two-site model, and the experiments and AW modeling of surface-reaction filamentary patterns (induced by a chain of dielectric breakdowns) are presented. Part II here continues with properties of AW (Sec. 2); and applications in nonliving systems (Sec. 3) and living systems — including those from social sciences and complex systems (Sec. 4), as well as in human history (Sec. 5). A comment on the relationship between physics, social science and complex systems is provided in Sec. 6. The review concludes with open problems in the form of workable research projects (Sec. 7) and general discussions (Sec. 8).

2. Properties of Active Walks

Analytical results for active walks are rare. An example is shown in Sec. 4 of Part I for the case of a two-site AW model. Most other results are obtained from computer simulations or numerical solutions of some coupled differential equations. An active walker may have a finite size. However, in all the studies published, the walker is treated as a point particle.

2.1. *Active walkers without internal states*

In this subsection, the point particle does not possess any internal state.

2.1.1. *Tracks and landscape surfaces*

Being a walker, the characteristic of a single active walker's track is of primary interest. Unlike a random walker which will walk forever, depending on how the stepping rule of the active walker is specified, the active walker may be trapped forever and "die" in a local minimum in the landscape potential V as in the case of the Probabilistic Active Walk (PAW). On the other hand, the walker will walk forever in the Boltzmann Active Walk (BAW), where the probability for the walker to jump from site i to adjacent site j is given by $P_{ij} \propto \exp\{\beta[V(i) - V(j)]\}$. Here $V(i)$ is V at site i, and β is the inverse "temperature." (See Sec. 3.1 of Part I for the definition of PAW and BAW.)

Some tracks from the PAW are shown in Fig. 1. More are given in Figs. 2 and 8 of Part I. For the PAW, the end-to-end length R_e and the radius of gyration R_g of a two-dimensional (2D) walk as a function of time are found to obey power laws. But the exponent depends on the parameters used in the walk, and is not universal [Lam *et al.*, 1992; Freimuth & Lam, 1992].

For the BAW, the 1D walk was studied by Kayser *et al.* [1992] and Pochy *et al.* [1993], and the 2D walk by Huang *et al.* [2002a], respectively, whereas the landscaping rule used in these two studies differ from each other. In the 1D study on a lattice, the landscaping function $W(r)$ is given by $W(0) = W_0$, $W(\pm 1) = -1$, and $W = 0$ otherwise.

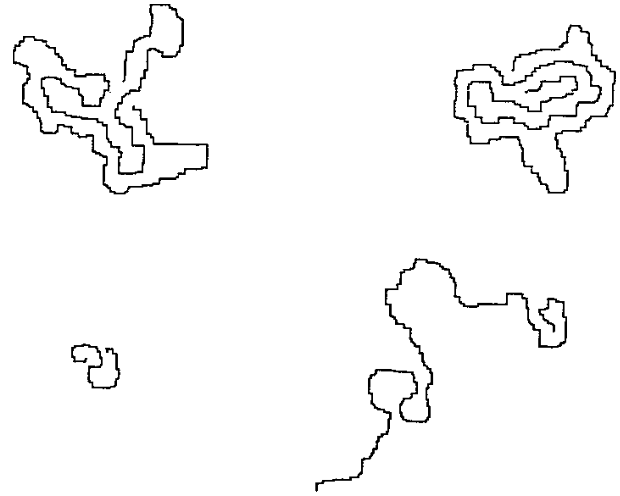


Fig. 1. Some tracks of a single walker from the PAW model [Freimuth & Lam, 1992].

The two parameters in this model are W_0 and β. Starting from a flat surface and after many steps of walks by the walker, the landscape V transforms into a fractal surface with its own unique scaling laws [Pochy *et al.*, 1993] (see Sec. 7.2). But the statistics of the tracks are not studied. A rather *sharp* localization-delocalization (LDL) transition is observed when W_0 is varied across the value of 2. For $W_0 < 2$, more surface is depleted around the walker than is being added to the walker's site. A groove of ever-increasing depth is formed and the walker is eventually self-trapped and localized. For $W_0 > 2$, the opposite happens; the walk is extended in space and the walker is delocalized. Scaling laws are found. This LDL transition was first reported in [Kayser *et al.*, 1992; Pochy *et al.*, 1993], and subsequently extended in [Lam, 1995a] and [Lam, 1997].

In the 2D study on a square lattice,

$$V(i) = -\{1 - \exp[-\alpha n(i)]\}, \tag{1}$$

where $\alpha > 0$; $n(i)$ is the number of times site i has been visited by the walker. Note that $V(i)$ can vary only between -1 and 0. Equation (1) represents the empirical law that a soft medium such as soil will deform under the impact of a load. This model also has two parameters, β and the stiffness exponent α. Typical morphology of the walks is shown in Fig. 2. Simulations of this model give two power laws,

$$R_e^2 = \langle R^2(t)\rangle \sim t^{2\nu}, \tag{2}$$

and

$$\langle S(t)\rangle \sim t^k, \tag{3}$$

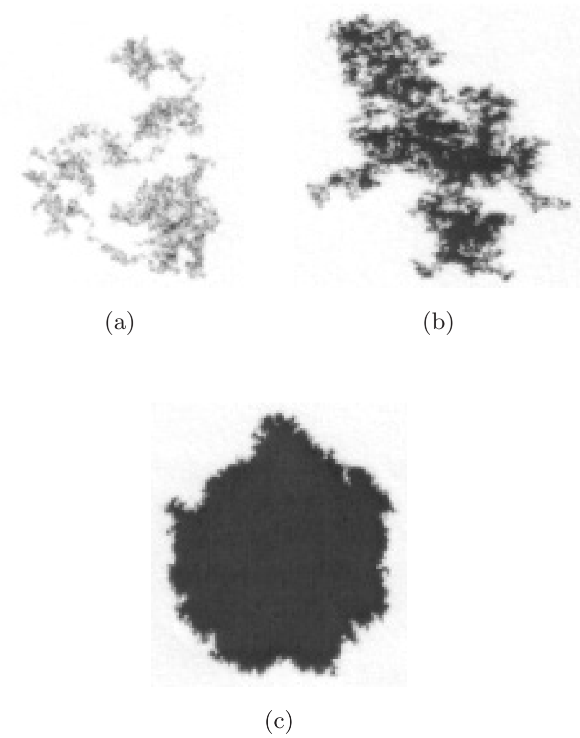


Fig. 2. Patterns of sites visited by a single walker in the 2D BAW model [Huang *et al.*, 2002a]. $\alpha = 1$ and $\beta =$ (a) 0.5, (b) 3, and (c) 6. The height of the landscape potential is represented by the gray scale, with darker gray indicating the more negative in height.

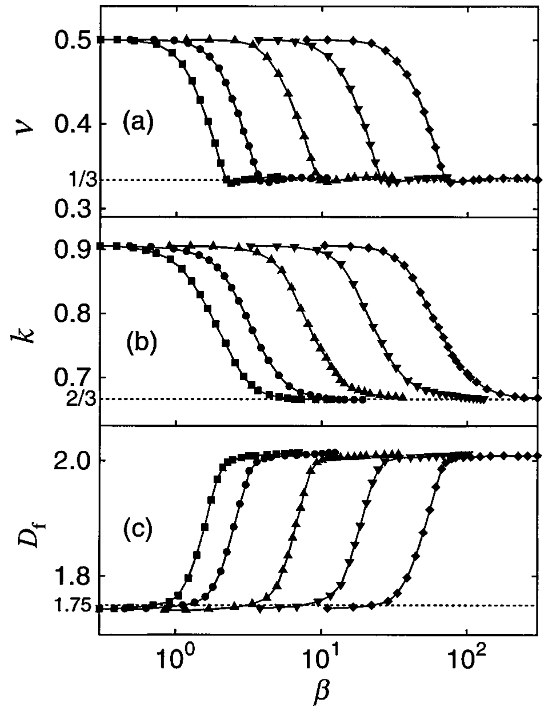


Fig. 3. Dependence of the three exponents ν, k and D_f on parameters α and β [Huang *et al.*, 2002a].

where $S(t)$ is the number of sites visited by the walker at time t, and the average is over many runs of the simulation. For a given α, as β increases from zero, ν decreases from 1/2 to 1/3 and k decreases from slightly above 0.9 to 2/3, as shown in Fig. 3, where also shown is D_f, the fractal dimension of the spatial distribution of sites visited. Physically, these results are easy to understand. The $\beta = 0$ limit of the BAW is the simple random walk, in which the walker does not feel the presence of the landscape, and $\nu = 1/2$ is expected. At $\beta = \infty$, the BAW becomes a deterministic walk such that the adjacent site most visited before will be chosen. The walk thus forms a compact cluster with $D_f = 2$. What is shown in Figs. 2 and 3 is thus a LDL transition, from an extended walk at $\beta = 0$ transiting to a compact walk at $\beta = \infty$. As can be seen in Fig. 3, the transition is not sharp. If somehow a β_c separating these two regions is defined (perhaps as the middle point in the transition region on the β axis), one finds $\beta_c = \exp(\alpha)$. All these results and other scaling laws are nicely explained in [Huang *et al.*, 2002a].

Multiwalkers were first studied in [Lam *et al.*, 1992; Freimuth & Lam, 1992]. The active walkers coexist on a common landscape. They deform the landscape individually, and communicate with and influence each other indirectly through the shared landscape. This scheme is able to reproduce and explain many of the patterns observed in nature, such as those in dielectric breakdowns, electrodeposits [Lam, 1995b] and retinal neurons. (See [Lam, 1997] for a review.) More recently, Huang *et al.* [2002b] extended their study of the 2D BAW model from a single walker to multiwalkers and obtained some interesting results. Initially, N ($= 1000$) active walkers are placed randomly on a flat surface, using a square lattice. They are randomly picked and allowed to move to an adjacent site according to the BAW stepping rule if that adjacent site is not occupied by another walker; otherwise the walker stays still. Every walker is given the chance to try in one iteration step. In the beginning, the landscape surface is essentially flat at zero height and the walkers are still far from each other. There is very little clustering of walkers when time t is small. At very large t, the whole area has been visited many times by the walkers and the whole surface is flat again at the lowest height of -1 everywhere. And the AW becomes a random walk for each of the walkers, and there is no clustering. Only at some intermediate t when the surface consists of valleys and hills that clustering of the walkers will occur, mainly at the valleys. Now let us define a clustering coefficient

$$\Gamma(t) \equiv \frac{\left\{\left[\frac{\sum_s s^2 n_s(t)}{\sum_s s n_s(t)}\right] - 1\right\}}{N-1}, \qquad (4)$$

where $n_s(t)$ is the number of clusters with s particles at time t. $\Gamma(t) = 0$ corresponds to no clustering; $\Gamma(t) = 1$, maximum clustering with all N particles forming a single cluster. As shown in Fig. 4, the $\Gamma(t)$ versus t curve obtained from simulation results of the BAW model indeed shows a single peak of value Γ^* at $t = t^*$. Both Γ^* and t^* are functions of α and β. These functions and the roughness of the surface are investigated in detail by Huang *et al.* [2002b].

2.1.2. *Clustering of active walkers*

In a multiwalker situation, clusters of active walkers are sure to form if each walker is able to deplete the landscape potential $V(\mathbf{r}, t)$ in its every step, and the walker has the ability to move around — excited by some noise, say — so that it will not be trapped permanently in some local minimum of V. Such a case was demonstrated by Schweitzer and Schimansky-Geier [1994] in a hybrid approach to the problem. They simulated the movement of the active walkers while a partial differential equation governing the evolution of V was solved numerically. Of course, these two components were coupled to each other in the model.

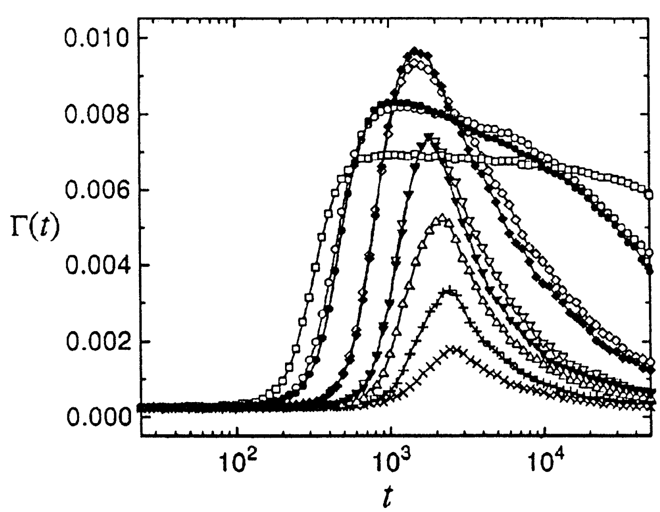


Fig. 4. Dependence of clustering coefficient $\Gamma(t)$ on time t. For all curves, $\alpha = 0.005$; β is fixed for each curve, varying from 3.16 to 10 from curve to curve [Huang *et al.*, 2002b]. (See Fig. 2 in [Huang *et al.*, 2002b] for more details.)

Specifically, a 2D triangular lattice is used and $V = 0$ initially. The walker likes to go from a high V position to a low V position, but not absolutely. It is a PAW of some sort. In the simulation, a walker checks the V in its six nearest-neighboring sites and compare with V at its present site, thus measuring the local potential gradient. If there is no attractive potential around, it makes a random choice and steps to one of these six neighboring sites. If there is an attractive potential, it goes to the adjacent site with the lowest potential, with a probability $1 - \eta$; and with probability η, it moves in a random direction. In other words, because of the noise (represented by η), the walker sometimes ignores the attraction of the potential and finds itself out of the local potential minimum. After all the walkers finish their one-step walk, V is updated by solving the equation

$$\frac{\partial V(\mathbf{r},t)}{\partial t} = D\nabla^2 V(\mathbf{r},t) - \gamma V(\mathbf{r},t) - \alpha p(\mathbf{r},t), \qquad (5)$$

where $p(\mathbf{r},t)$ is the probability density of the walkers, γ the damping coefficient, α a constant parameter, and D the diffusion constant of V. The negative sign on the last term on the right-hand side of Eq. (5) corresponds to the depletion of V due to the walkers. If the walkers are the ants, V will be the *negative* density of the pheromone released by the ants; an ant's preference to go to high pheromone density places corresponds to the walker going to low V places, as is assumed here. Numerical results indeed show clustering of the walkers, as depicted in Fig. 5.

The clustering effect can be understood as follows. Under the adiabatic approximation that $\partial V(\mathbf{r},t)/\partial t \approx 0$ and $D \to 0$, Eq. (5) gives

$$V(\mathbf{r},t) = -\left(\frac{\alpha}{\gamma}\right) p(\mathbf{r},t) \qquad (6)$$

Putting Eq. (6) into the Fokker–Planck equation that presumably is obeyed by p,

$$\frac{\partial p(\mathbf{r},t)}{\partial t} = D_w \nabla^2 p(\mathbf{r},t) + \nabla[p(\mathbf{r},t)\nabla V(\mathbf{r},t)] \qquad (7)$$

we obtain

$$\frac{\partial p(\mathbf{r},t)}{\partial t} = \nabla[D_{\text{eff}}(\mathbf{r},t)\nabla p(\mathbf{r},t)]. \qquad (8)$$

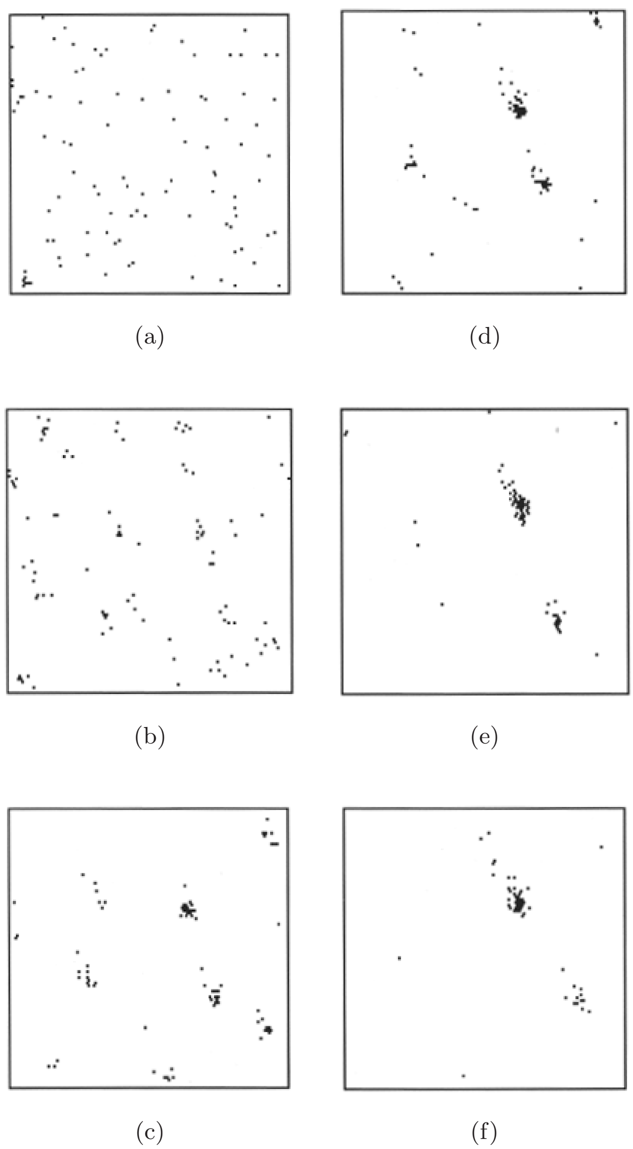


Fig. 5. Clustering of active walkers [Schweitzer & Schimansky-Geier, 1994]. The positions of the walkers are shown in (a)–(f), as time increases.

Here, the effective diffusion constant is given by

$$D_{\text{eff}}(\mathbf{r},t) = D_w - \left(\frac{\alpha}{\gamma}\right) p(\mathbf{r},t). \tag{9}$$

When $p(\mathbf{r},t)$ is large enough, we may have $D_{\text{eff}}(\mathbf{r},t) < 0$ — clustering at location $\mathbf{r}$. Note that in this case, approximations are made so that the two coupled equations, Eqs. (5) and (7), are reduced to a single equation, Eq. (8). The original coupled equations are not really solved.

The clusters appear where $V(\mathbf{r},t)$ has a large dip. When $-V$ is plotted in the 2D $\mathbf{r}$ space, these dips show up as spikes. The spikes corresponding to the clustering process of Fig. 5 go through two stages. In the beginning, these spikes appear individually; then as larger clusters appear, only a few large spikes are left.

However, if Eq. (7) is modified to become

$$\frac{\partial p(\mathbf{r},t)}{\partial t} = D_w \nabla^2 p(\mathbf{r},t) + \nabla[\,p(\mathbf{r},t)\nabla V(\mathbf{r},t)] - Ap(\mathbf{r},t) + B, \tag{10}$$

where the A and B terms represent desorption and adsorption of the active walkers, respectively, due to external influence, then stable coexistence of spikes — almost periodically arranged in space — is possible [Schimansky-Geier *et al.*, 1995].[1]

2.1.3. *Spirals and traveling spots*

In excitable systems, coupled reaction–diffusion systems [Walgraef, 1997] are known to be able to produce spirals and traveling localized spots. The same can be reproduced more efficiently by the use of active walkers [Schimansky-Geier *et al.*, 1995].

For example, the two coupled equations could be given by Eq. (5) and

$$\frac{\partial p(\mathbf{r},t)}{\partial t} = D_w \nabla^2 p(\mathbf{r},t) - kp(\mathbf{r},t) + C(1+V(\mathbf{r},t))\Theta(p-p_c), \tag{11}$$

where the Heaviside function is defined by $\Theta(x) = 1$ if $x > 0$, and $\Theta(x) = 0$ if $x < 0$. Here, in comparison with the case in Eq. (7), the active walkers are no longer influenced by the gradient of V, but replicate if its population exceeds a threshold p_c; and a decay term is added. This is an activator–inhibitor system, with the active walkers acting as activator, and V the density of an inhibitor. The hybrid approach is again used, i.e. Eq. (5) is solved numerically, but Eq. (11) is replaced by the simulation of 10^4 active walkers. When p_c is a constant, target waves, single and double spirals are obtained. When p_c is assumed to linearly increase with the global number of active walkers, traveling spots are formed instead.

2.1.4. *Other properties*

Chance versus necessity is an important issue in growth problems. Chance, in the form of noise or the series of random numbers used in a computer

[1] In [Schimansky-Geier *et al.*, 1995], active walkers are called "active Brownian particles."

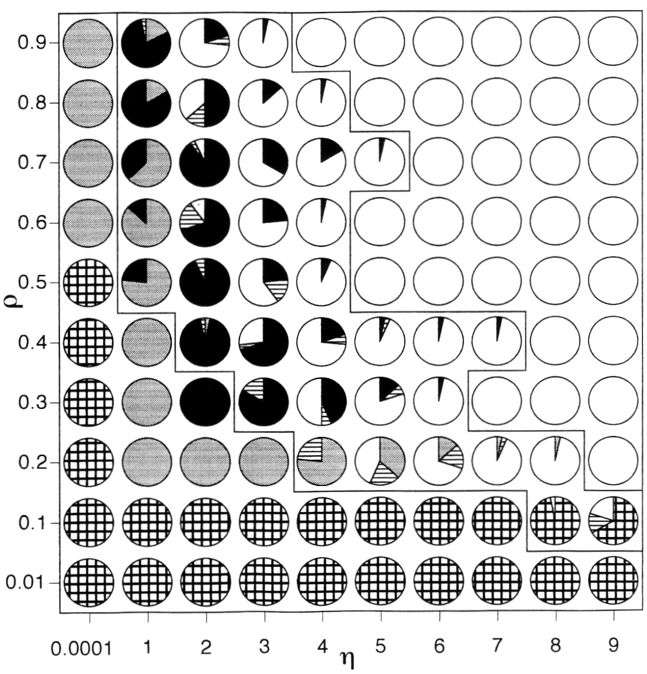


Fig. 6. Morphogram in the (η, ρ) plane from the active-walk aggregation model [Lam *et al.*, 1995]. η and ρ are two control parameters in the probabilistic model. The pie chart at each point represents the percentage of each type of five possible morphologies obtained from 30 runs of the algorithm. Within the "sensitive zone" in the middle region — where chance plays a dominant role — the outcome is unpredictable.

algorithm, is shown to be important sometimes but not always, as demonstrated in the active-walk aggregation model [Lam *et al.*, 1995; Lam *et al.*, 1996]. (See Fig. 6.)

The two types of intrinsic abnormal growth — transformational growth and irreproducible growth, as illustrated by AW models, are summarized in [Lam, 1997].

2.2. *Active walkers with internal states*

Let the active walker possess an internal state θ so that $\theta = 0$ or 1 (like particles with two possible colors). And let us say that only those walkers with $\theta = 1$ can interact with the potential V, identified as a temperature distribution. Assume a constant number of walkers present, but the two kinds of walkers can interchange into each other with given rates — the change rate from $\theta = 1$ type to $\theta = 0$ type is temperature dependent, while the reverse change rate is a constant. Under suitable conditions, a traveling wave in 1D is found [Schimansky-Geier *et al.*, 1997]. This example shows the interesting possibilities of active walkers with internal states. More examples are given in [Schweitzer, 2003].[2]

3. Applications of Active Walks in Nonliving Systems

A few important applications of AW in nonliving systems are given here; those in living systems will be presented in the next section. Unlike those in Sec. 2, the AW models studied here and in Sec. 4 are closely linked to real experimental data.

3.1. *Filamentary patterns in electrodeposits and surface reactions*

Filamentary patterns are naturally modeled by AW, since the track of an active walker is a filament. Applications of AW to experiments in electrodeposits and in surface-reaction filaments induced by dielectric breakdowns are reviewed in [Lam, 1995b, 1997] and Sec. 5 of Part I.

3.2. *Compaction in granular matter*

Granular matter is in the forefront of research today [Duran, 2000; Chen *et al.*, 2002]. They are neither fluids nor solids; a proper theory for these strange materials is still lacking [Kadanoff, 1999]. The connection between AW and granular matter is discussed in [Lam, 1997].

In a compaction process, the granular medium reorganizes into "domains." After an initial transient, compaction proceeds as a coarsening for the domains and a progressive reduction of domain boundaries. A model of active walkers moving on active substrates is proposed by Baldassarri *et al.* [2001] to explain the coarsening process. Assuming that the system is uniform in the direction of gravity, the medium is described by a 1D density profile $\rho(x,t)$, in which the domain wall — treated as an overdamped active walker located at $x = X(t)$ — moves. The coupled equations are

$$\frac{dX}{dt} = -\int dx\delta(x - X(t))\frac{\partial V[\rho(x,t)]}{\partial x} + \Gamma(t), \quad (12)$$

and

$$\frac{\partial \rho(x,t)}{\partial t} = f(\rho(x,t))\delta(x - X(t)), \quad (13)$$

[2]Active walkers in [Schweitzer, 2003], except those in Chapter 5, are called "Brownian agents." Note that active particles in Chapter 2 there are particles with one internal state; they are not coupled to a landscape, and hence are not active walkers.

where $\Gamma(t)$ is an uncorrelated Langevin force. The potential V attracts the walker to regions where activity has been intense, and repels it from unvisited regions. Physical considerations lead to the form $f(\rho) = (1-\rho)^a$, and $V(\rho)$ behaving linearly in ρ as $\rho \to 1$. Under these conditions, Eqs. (12) and (13) are recast as

$$\frac{dX}{dt} = F^{-1-b}\nabla F + \Gamma(t), \tag{14}$$

and

$$\frac{dF}{dt} = \delta(x - X(t)), \tag{15}$$

where $b = 1/(a-1)$, and $F(\rho) = (a-1)^{-1}(1-\rho)^{1-a}$. As it stands, the landscape in Eqs. (14) and (15) is represented by F. The consequence of this model is that the high density regions (i.e. the potential wells) tend to trap the walkers that, in turn, are able to change the environment, the local density, though their efficiency in doing so decreases with the increase of the density — resulting in a drastic slowing down. The way the walker escapes from the potential well is to progressively carve their way out by pushing the potential barrier and so enlarging the compactified region.

3.3. *Applications in geology*

Geological systems provide a fertile land for studying self-organizing processes that, quite often, manifest themselves in the form of spatiotemporal patterns. Some of these patterns are fractals [Turcotte, 1997; Rodriquez-Iturbe & Rinaldo, 1997]; some are not. In many cases, the formation process involves the change of the geological landscape. Active walk consists of particles sculpturing a deformable landscape, and is the natural choice in modeling the dynamics of these geological phenomena.

Cold production in oil recovery is a nonthermal process in which sand production is encouraged and high permeability channels are formed. These channels grow into a branched wormhole network when water is forced into the ground in a shaft. A mixture of sand and oil moves up the channels to the ground surface, with the proportion of oil increasing at later stages. The wormhole tip grows as the liquid flux erodes the soil, a local process appropriately modeled by AW. Indeed, assuming that the diameter of the wormhole channel decreases with a power law as it grows, the AW model does describe excellently this oil recovery process [Yuan *et al.*, 1999].

River basins are formed when rain falls, runs downhill and erodes the landscape. River channels emerge and branch due to the original topology of the landscape but also the erosion process caused by the flowing water. The erosion process can be represented by the action of an active walker [Lam & Pochy, 1993].

3.4. *Active walk model for experimental parameter-tuning networks*

In a large apparatus made up of many components, each component needs be tested and tuned separately and then collectively, before experiments using the apparatus as a whole can be conducted. And there are a large number of parameters to be tuned. How can we model the adjustment process of these parameters and make some sense out of it? In this subsection, the ideas, formulations and results due to Han [2005] are reported.

In a Tokamak in plasma physics, in the ion source segment of the neutral-beam injector system alone, there are eight major parameters to be adjusted. The experimenter sets the parameter values, turns on the equipment, and measures the outcome of a certain quantity Q. If the Q obtained does not meet the mark Q_0, say, the whole process is repeated, with a new set of parameters. The adjustment process stops when Q is equal or very close to Q_0. The simultaneous adjustment of a large number of parameters is a complicated process, which is based on the feedback from previous Q values obtained, the experimenter's experience, and the limitation of the hardware that control the parameters.

3.4.1. *Construction of parameter-tuning networks*

To quantify this complicated process, a parameter-tuning network can be constructed as follows. To illustrate the idea in detail, we assume here that only three parameters u_1, u_2 and u_3 are involved. The experimenter's serial adjustment of $\mathbf{u}[\equiv (u_1, u_2, u_3)]$ can be represented by a sequence of connected dots in the 3D $\mathbf{u}$ space, as shown in Fig. 7. For each choice of $\mathbf{u}$, Q is measured, so that $Q = Q(\mathbf{u})$ but the functional form is unknown due to the complexity of the equipment, which is like a black box.

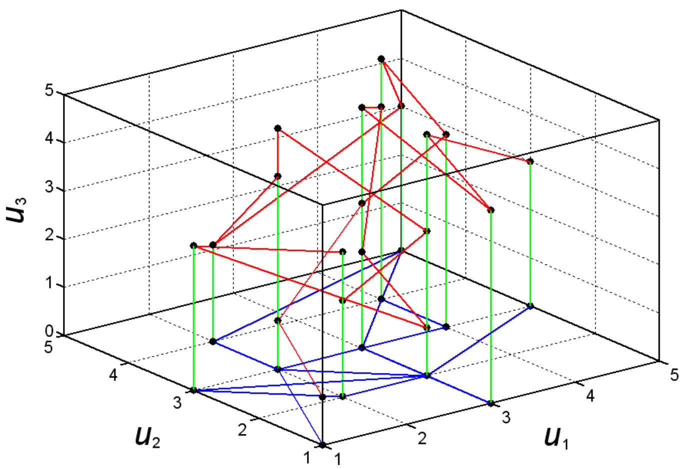
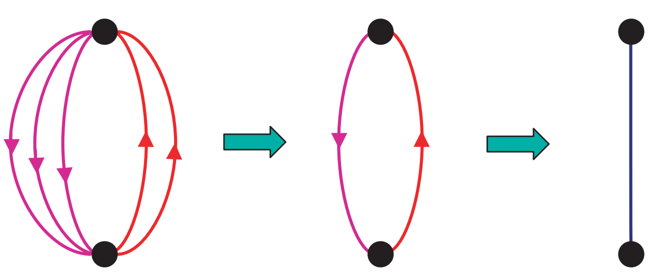


Fig. 7. The sequence of three tuning parameters created through the action of an experimenter who changes the parameters in some order, in a sequence of imagined experiments using the same piece of equipment. The red lines linking the dots show the sequence, with the direction of the sequence suppressed in this drawing. The blue lines are the projection of the red lines on the (u_1, u_2) plane. The green lines show how the red lines are projected.

The sequence of **u** dots is then projected onto the (u_1, u_2) plane. The dots on the same vertical line along the u_3 direction are allowed to collapse to one point, called a node. There are 12 such nodes in Fig. 7. A line connecting two nodes is called an edge [Albert & Barabási, 2002]. Since in real situations, the parameters selected by the experimenter in different adjustments may partially overlap with each other, there may be more than one edge connecting two nodes, as indicated in Fig. 8(a). These edges are directed. For simplicity, we make the approximation of going from (a) to (b) in Fig. 8 by collapsing all the edges between two nodes that have the same direction as one single edge with the same direction. Consequently, between two nodes, there are at most two edges with opposite directions, as indicated in Fig. 8(b), resulting in a *directed* parameter-tuning network. The directed network thus formed from Fig. 7 is shown in Fig. 9.

A *nondirected* network is formed from the directed one, by going through (b) to (c) in Fig. 8. Figure 10 shows the nondirected network derived from that in Fig. 9.



(a) (b) (c)

Fig. 8. Simplifications adopted in forming the parameter-tuning network. The first step is from (a) to (b), with edges of the same direction collapsed into a single directed one, resulting in a directed network. The second step is to collapse the two edges in (b) into one in (c) and ignore the directions, leading to a nondirected network. [Note that occasionally, we may have only one set of edges with same direction in (a) and (b).]

3.4.2. *Counting the edges*

In Fig. 9, for node $j (j = 1, 2, \ldots, N_n$, with $N_n = 12)$, let $N_+(j)$ be the number of in-going edges, and $N_-(j)$ the number of out-going edges. In Fig. 10, let $N(j)$ be the number of edges at node j. The results are given in Table 1. Note that $N(j) = N_+(j) + N_-(j)$ is not always true; the untrue cases are $j = 3$ and 5.

3.4.3. *Degree distribution functions*

Now count the number of N_+ in Table 1 that has the value k, and call it $p_+(k)$; similarly for $p_-(k)$

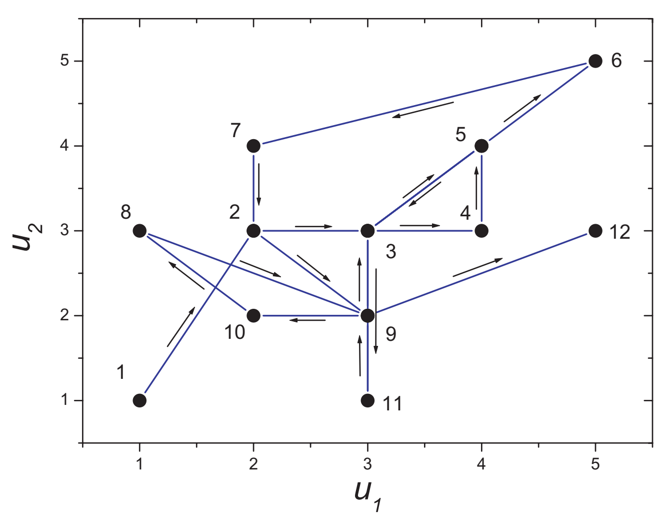


Fig. 9. The directed network formed by projecting the red segments in Fig. 7 onto the (u_1, u_2) plane, after the simplification of going from (a) to (b) in Fig. 8. Each arrow indicates a directed edge. There is an edge connecting node 3 directly to node 11, in the direction shown by the long arrow. This edge is treated as a separate edge, and should not be confused with the two short edges from node 11 to node 9, and from node 9 to node 3, respectively. The total number of nodes in this diagram is N_n, which is equal to 12.

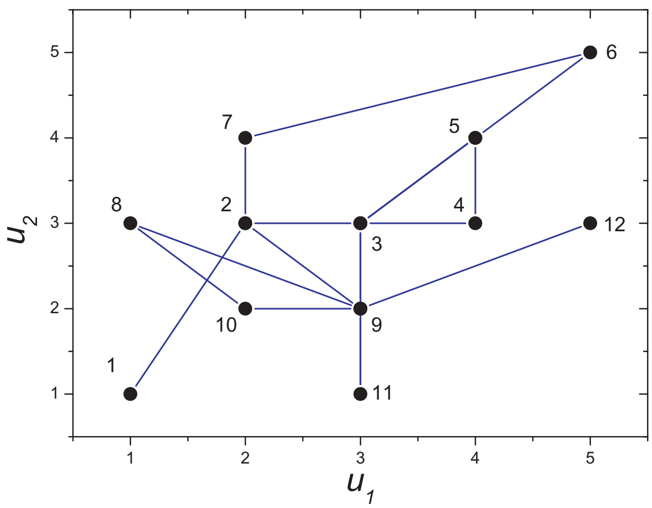


Fig. 10. The nondirected network as derived from the directed network of Fig. 9.

Table 1. Counting the edges. N_+ and N_- are read from Fig. 9, N from Fig. 10.

j	$N_+(j)$	$N_-(j)$	$N(j)$
1	0	1	1
2	2	2	4
3	3	3	5
4	1	1	2
5	2	2	3
6	1	1	2
7	1	1	2
8	1	1	2
9	3	3	6
10	1	1	2
11	1	1	2
12	1	0	1

Table 2. Degree distributions and cumulative degree distributions.

k	$p_+(k)$	$p_-(k)$	$p(k)$	$P_+(k)$	$P_-(k)$	$P(k)$
1	7	7	2	11	11	12
2	2	2	6	4	4	10
3	2	2	1	2	2	4
4	0	0	1	0	0	3
5	0	0	1	0	0	2
6	0	0	1	0	0	1

and $p(k)$. k is called the "degree" of a node. $p_+(k)$ is called the "in-degree distribution," $p_-(k)$ the "out-degree distribution," and $p(k)$ the "degree distribution." The results are shown in Table 2.

Next define the corresponding "culmulative degree distributions" $P_+(k)$, $P_-(k)$ and $P(k)$ by

$$P_+(k) \equiv \sum_{k'=k}^{k_m} p_+(k') \tag{16}$$

$$P_-(k) \equiv \sum_{k'=k}^{k_m} p_-(k') \tag{17}$$

$$P(k) \equiv \sum_{k'=k}^{k_m} p(k') \tag{18}$$

Here k_m is the maximum k that the p function is nonzero. $k_m = 3$ for Eqs. (16) and (17), and $k_m = 6$ for Eq. (18). The results are shown in Table 2. Note that by definition, the P's are monotonic decreasing functions, while the p's may not be monotonic at all. The reason for introducing the P's is that we want to fit them to monotonic decreasing functions such as stretched exponents or power laws. We are not going to plot the data from Table 2; instead, we will do that for data from real experiments and an AW model.

3.4.4. *Experimental results*

In real experiments studying the ion source of neutral beam injector system for HT-7 Tokamak, eight control parameters are involved. These include the filament current, magnet current, arc voltage, cathode gas valve voltage, anode gas valve voltage, etc. Each of the parameters has 10 to 30 discrete, adjustable setting values. The aim of the experiment is to find out which set of parameters will give a strong and stable discharge, measured by the arc current intensity Q. Each parameter is set by a dial which can be turned left or right between two extreme positions. When the extreme position is reached, the experimenter has to turn the dial back, reversing the direction of turning.

From the experimental data, diagrams like Figs. 7, 9 and 10 are generated, except the **u** space now assumes eight dimensions. The P's are plotted and fitted nicely with stretched exponent functions (Fig. 11) such that

$$P(k) \sim \exp(-ak^\gamma), \tag{19}$$

or, equivalently,

$$\ln P(k) = -ak^\gamma + b, \tag{20}$$

and similarly for $P_+(k)$ and $P_-(k)$.

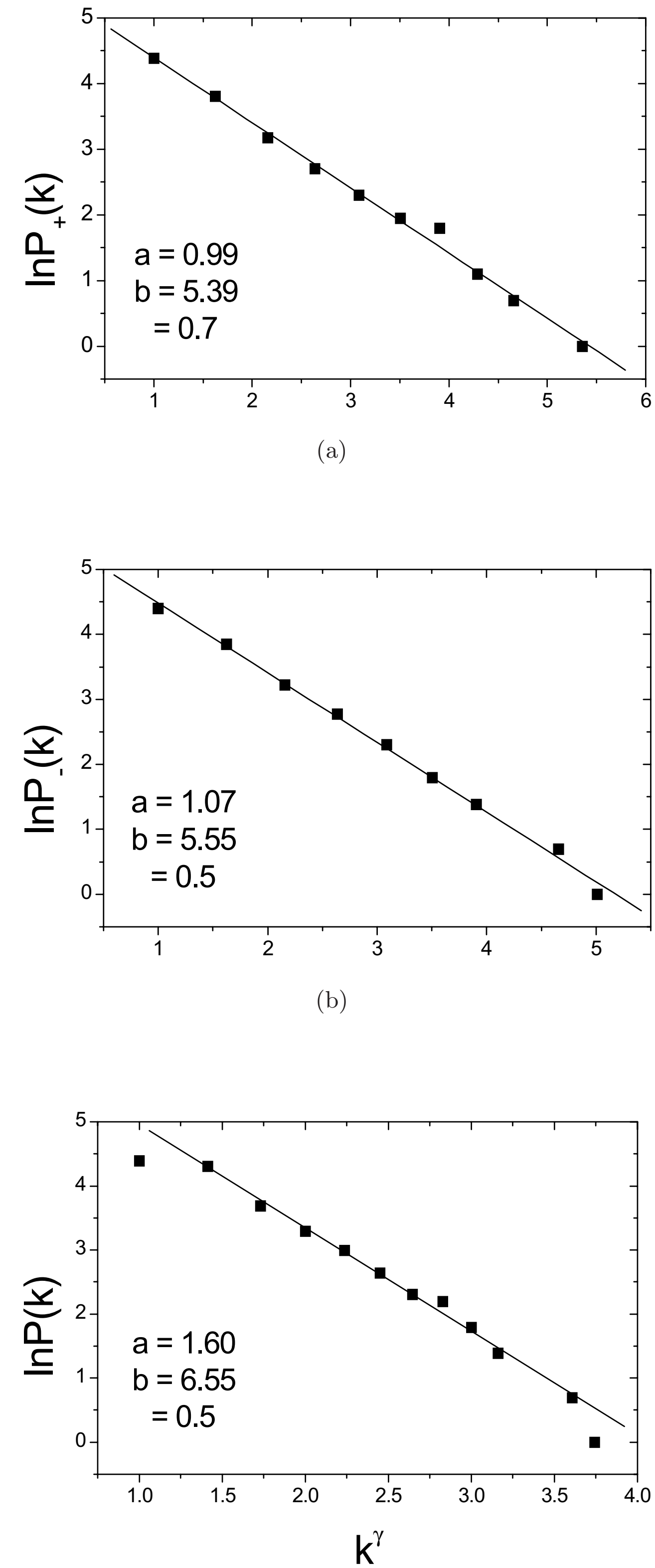


Fig. 11. Experimental curves of $P_+(k)$, $P_-(k)$ and $P(k)$, fitted to stretched exponent functions. The two corresponding (directed and nondirected) parameter-tuning networks exist on the (V, I) plane, where V is the cathode gas valve voltage and I the filament current.

3.4.5. *Active walk modeling*

The adjustment of the parameters is kind of correlated by the experimenter, and the process is modeled by an AW model. For demonstration, we assume the simplest case of three parameters, $\mathbf{u} = (u_1, u_2, u_3)$. Each u_i has adjustable values of $1, 2, \ldots, 20$, say. The adjustment of the ith parameter is represented by the movement of an active walker on a 1D landscape potential $V_i(x_i)$ $(i = 1, 2, 3)$. The allowable x_i for the ith walker are the integers, $(1, 2, 3, \ldots, 100)$, the same for all i. These 100 numbers are mapped to the u_i, such that if the ith walker ends in the region [1, 5] on the x_i axis, u_i will assume the value of 1. Similarly, the region of [6, 10] is mapped to $u_i = 2$, etc. This mapping can be attained by

$$u_i(t) = \mathrm{Int}\left[\frac{X_i(t) - 1}{5}\right] + 1, \tag{21}$$

where $X_i(t)$ is the position of the ith walker on the x axis at time t, and Int means taking the integral part of the number. This mapping of x_i to u_i has the effect of making two consecutive sets of adjusted parameters more likely to partially overlap with each other, and ensures the occurrence of smooth V_i versus x_i curves. The x_i could be taken simply as u_i if the u_i range is large.

In the simulations, all V_i start flat at time $t = 0$, and are updated simultaneously at each time step $t(= 1, 2, \ldots)$, according to a rule to be specified below. But between two consecutive time steps, each walker moves a few steps in a subwalk on its own V_i, independent of other walkers. In a subwalk, the walker does *not* change V_i. The subwalk is like a particle rolling on a landscape with friction, with the following rules (with the subscript i removed for the sake of clarity). The subwalk time is labeled by $\tau(= 0, 1, 2, \ldots)$.

1. At time $t = 0$, the particle is arbitrarily placed on the x axis.
2. At time t, the particle is given energy K_0 at $\tau = 0$. (K_0 is a parameter fixed in the model.)
3. At subwalk time τ, the particle moves left or right with equal probability. However, after each move, the particle loses energy ε, and the energy difference $V(x(\tau + 1)) - V(x(\tau))$ — which could be positive or negative — is added to its energy. Consequently, at time τ, the particle already moves τ steps, and its energy becomes

$$K(\tau) = V(x(\tau)) - V_0 + K_0 - \tau\varepsilon, \tag{22}$$

where $V_0 \equiv V(x(0))$, the potential at the initial position of the particle at $\tau = 0$.

4. The particle can get over a potential barrier in V that is lower than K, and can rebound if the barrier is higher than K.
5. When the particle reaches the left boundary ($x = 1$) or the right boundary ($x = 100$), it reverses direction and continues its subwalk.
6. The particle stops when it walks into a potential well and cannot get out with its energy K, or exhausts its energy on a plateau.

These possibilities are sketched in Fig. 12. The reflecting boundary condition in item 5 corresponds to the real experimental situation where a dial with a limited range is used.

After all three particles stop, the time clock increases from t to $t+1$; the particle's position at the end of its subwalk is taken to be $X(t+1)$, which is the starting position of the subwalk at time $t+1$. (The subwalks of the particles may not stop after the same number of subwalk steps; those stop first will sit there and wait for the last particle to stop.) The landscape $V_i(x_i)$ is updated by the landscaping rule:

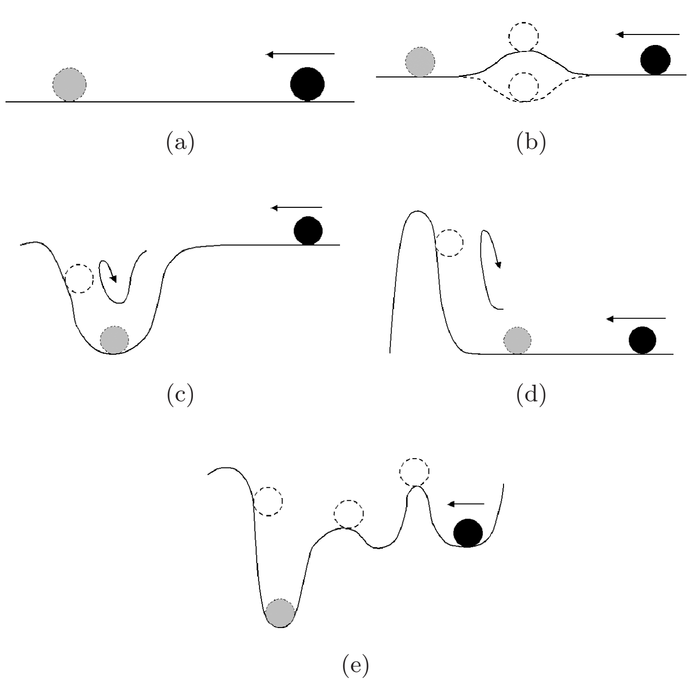


Fig. 12. Sketch of possible subwalks of a particle. The solid dot, gray dot and open circle represent the initial position, final position and some intermediate position of the particle, respectively; the arrow indicates the initial direction of the particle. (a) The particle moves and stops on a plateau. (b) The particle gets over a low potential barrier or a shallow well. (c) The particle stops in a deep well. (d) The particle rebounds on a high potential barrier. (e) The particle moves on a general landscape, and ends up in a deep well after many steps.

$$V_i(x_i; t+1) = \begin{cases} V_i(x_i;t) + W(x_i - X_i(t+1)), & E(t) \geq E_a(t) \\ V_i(x_i;t) - W(x_i - X_i(t+1)), & E(t) < E_a(t) \end{cases} \tag{23}$$

where $t = 0, 1, 2, \ldots$ For example, in numerical simulations below, the (i-independent) landscaping function W is given by $W(0) = 4$, $W(\pm 1) = 3$, $W(\pm 2) = 1$, and $W = 0$ otherwise. In Eq. (23), the error function $E(t)$ is assumed to be

$$E(t) = |Q(\mathbf{u}(t)) - Q_0| \tag{24}$$

where $\mathbf{u}(t)$ is obtained from Eq. (21); $E_a(t)$ is the average error counting the last n time steps, given by

$$E_a(t) \equiv \frac{1}{n} \sum_{t'=t-n+1}^{t} E(t'). \tag{25}$$

In computer simulations, we pretend that we know Q_0 and the $Q(\mathbf{u})$ function. In reality, the former is known to the experimenter; the latter can be roughly inferred from experimental data. Note that while the subwalks of the particles are independent of each other, the updating of their landscapes is affected by their collective effort through $Q(\mathbf{u})$ in $E(t)$.

3.4.6. *Simulation results and discussion*

The three-parameter AW model is simulated with t going from 0 to 1000. The function Q is assumed to be

$$Q(\mathbf{u}) = |u_1 - 10| + |u_2 - 10| + |u_3 - 10|. \tag{26}$$

The parameters used are: $K_0 = 23$, $\varepsilon = 1$, $n = 5$ and $Q_0 = 0$. This means that $\mathbf{u} = (10, 10, 10)$ is the one and only optimal parameter set.

The landscape functions at various time t are given in Fig. 13. The error function $E(t)$ is displayed in Fig. 14, which shows that the optimal $\mathbf{u}$ is first obtained at $t_0 = 122$. The last 700 $\mathbf{u}(t)$ dots (with $t > 300$) in the $\mathbf{u}$ space are projected onto the (u_1, u_2) plane to obtain the directed and nondirected networks; the latter is shown in Fig. 15. The corresponding cumulative degree distributions are given in Fig. 16. Good stretched-exponent fits are

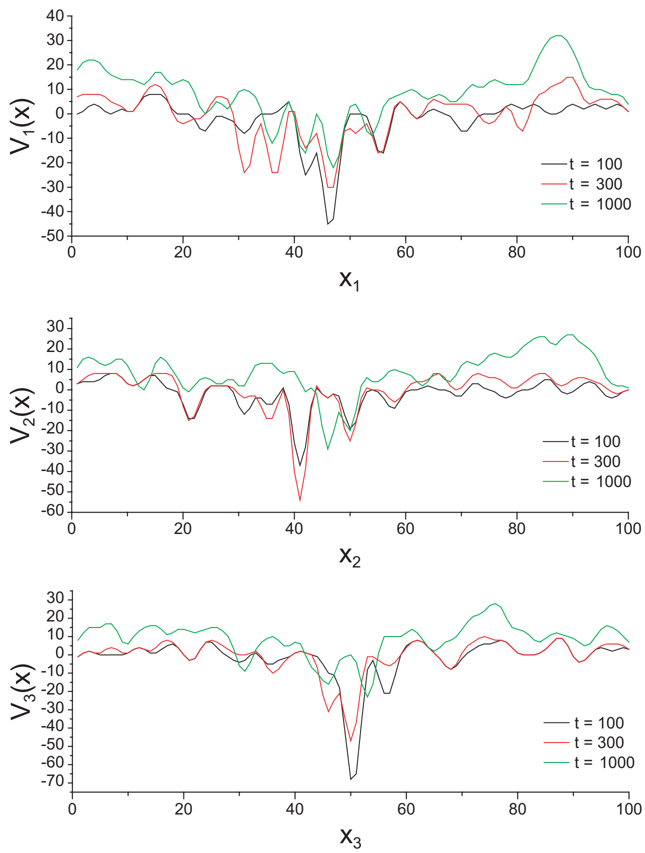


Fig. 13. Time evolution of the landscape potentials.

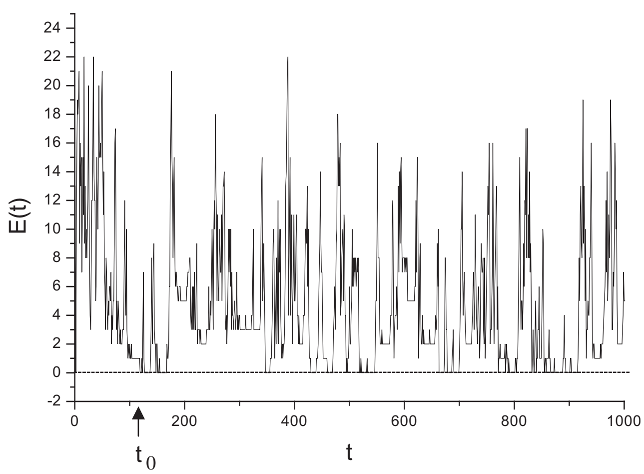


Fig. 14. Dependence of the error function on time.

obtained, in agreement with the experimental findings in Fig. 11.

The subwalks somewhat mimick the action of the experimenter — that is why the AW model and the experiments give similar P functions. Stretched exponent distributions exist in other networks [Xulvi-Brunet & Sokolov, 2002; Holanda *et al.*,

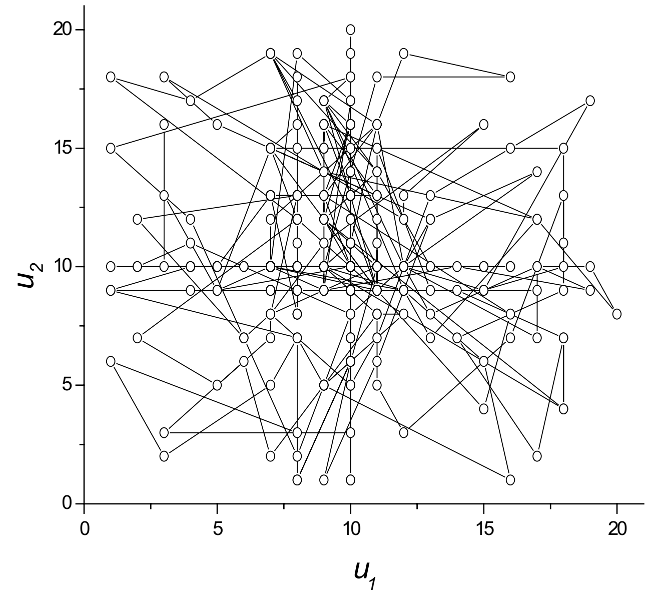


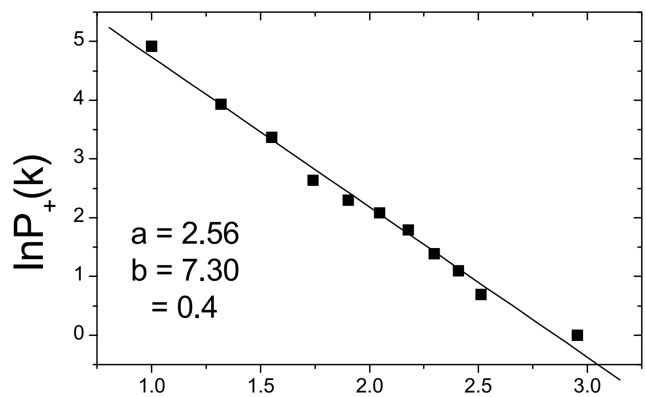
Fig. 15. The nondirected parameter-tuning network obtained from the simulation of a three-parameter active walk model. The two (directed and nondirected) networks exist on the (u_1, u_2) plane.

2004]. The important problem is to understand the mechanisms that give rise to these stretched exponents, and what they mean. By modifying the subwalks and tuning their parameters one may shorten t_0, the shortest time to find the optimal parameters, which would be of interest to the experimenters. In fact, the AW model here could be developed into a general method to tackle optimization problems in many other systems.

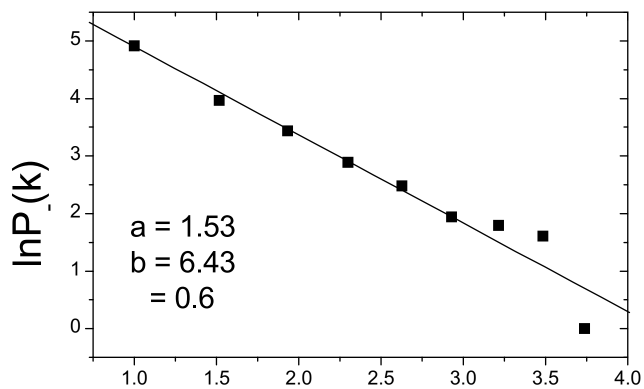
3.5. *Other applications*

Anomalous ionic transport in glasses is an interesting application of AW (see [Lam, 1997]). Another example is the self-consistent coupling of laser with a dielectric in nonlinear optics.

When a light beam propagates in a nonlinear medium, the photon's path is influenced by the local refractive index of the medium, which in turn is changed by the photon as it propagates [Shen, 2002]. The photon is thus an active walker; the landscape is the distribution of the refractive index. But one need not start from the photons. For instance, in a photorefractive medium, spatial screening solitons can be treated as rays using geometrical optics [Belić *et al.*, 2000]. The ray picture is transformed into a classical mechanics picture, within which solitons move self-consistently as particles in a potential created by the induced change in the refractive index.



(a)



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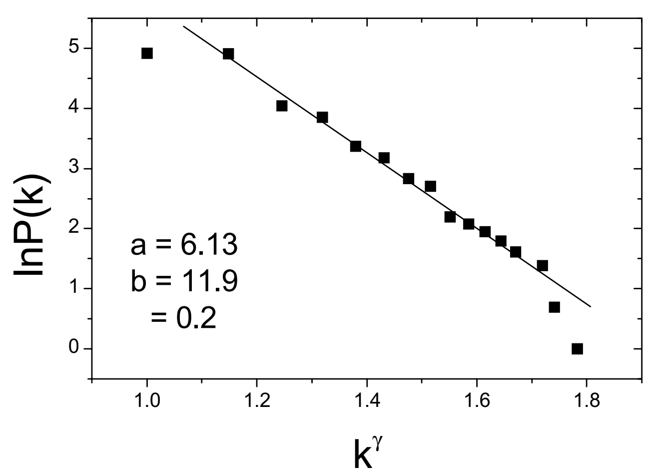


(c)

Fig. 16. Numerical curves of $P_+(k)$, $P_-(k)$ and $P(k)$ obtained from the parameter-tuning networks from a three-parameter active walk model simulation, fitted to stretched exponent functions.

4. Applications of Active Walks in Living Systems

Living systems include ants, bacteria and humans. Humans are studied in social science and in history. Applications of AW in social science[3] are included in this section, while those in history are delayed to the next section.

4.1. *Spontaneous formation of human trails and ant trails*

Human trails, either on soft soil or grass, are formed spontaneously, not purely by chance, but by a combination of the locations of places the pedestrians want to go, from where, and practical considerations of duration and convenience. In the AW model of Helbing *et al.* [1997a],

1. Deformation of grass (in a campus or an urban park) due to footsteps of pedestrians is assumed to stay for a finite amount of time.
2. The landscape is a potential function $V(\mathbf{r}, t)$ determined by the depth of footprints within a circle around space point $\mathbf{r}$.
3. The walker chooses the next step according to the combined effect of taking the shortest distance to the destination and the affinity to existing footprints around it (by going to where V is lower).

In this work, a set of coupled partial-differential equations is written down and solved numerically, even though the simulation can be done without them. The results match qualitatively and impressively well with what are observed in the real world. An example is shown in Fig. 17.

Ants as active walkers are pointed out by Lam and Pochy [1993]. Ant trails with spontaneous branching laid down by ants foraging for food are successfully simulated with active walkers [Schweitzer *et al.*, 1997]. More details are given in [Helbing *et al.*, 1997b].

4.2. *Bacteria pattern formation*

The growth of bacteria by migration and division in "friendly" conditions yield colonies of simple compact patterns. Under hostile conditions created in a petri dish with very low nutrients, a hard surface, or both, complex patterns are often observed

[3]The applicability of active walk in modeling social systems is recognized by Batty [1997], among others.

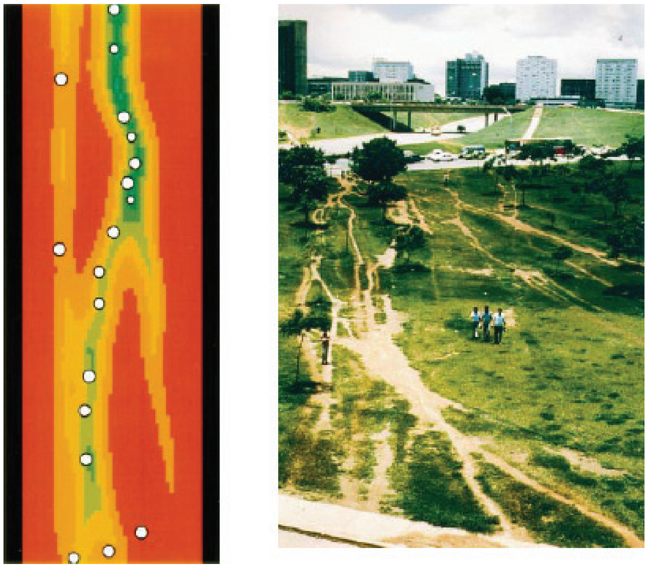


Fig. 17. Pedestrian trails [Helbing *et al.*, 1997a]. Active walk simulation result (left) and the trail system on the university campus of Brasilia (right).

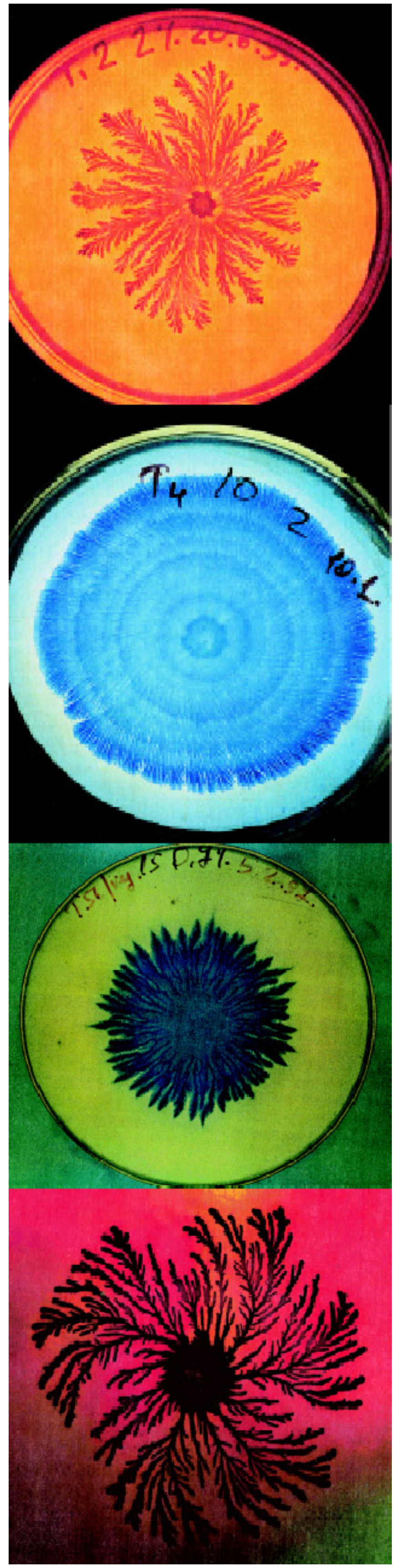


Fig. 18. Examples of bacteria patterns formed under hostile conditions [Ben-Jacob *et al.*, 2000].

[Ben-Jacob *et al.*, 2000]. Some examples are shown in Fig. 18. Bacteria patterns similar to those found in nonliving systems were first reported by Matsushita's group in 1989 [Matsushita *et al.*, 1995].

Bacteria are sophisticated entities that can detect the nutrient concentration near it, receive and send out chemical signal, and move accordingly. They are best modeled as active walkers with internal states, moving in a 2D space. In the work of Kessler and Levine [1993], the nutrient concentration $c(\mathbf{r}, t)$, the landscape, is given by

$$\frac{\partial c}{\partial t} = D\nabla^2 c - \gamma c + (\text{sources}), \qquad (27)$$

which is solved numerically. Here, the source term represents contributions from the active walkers, called "bions" by the authors. Each walker at location $\mathbf{R}$ has internal states such that

1. The walker remains in state 0 until it detects $c(\mathbf{R}) > c_T$, a fixed threshold.
2. Once $c(\mathbf{R}) > c_T$, the walker changes to excited state 1 and emits an amount Δc over τ time units. Δc contributes to the source term in Eq. (27) — the landscaping action due to the walker.
3. After time τ, the walker changes to quiescent state 2 and counts until time t_R units before it reverts to state 0. Until it reverts, it is immune to further excitation.

The stepping rule of the walker is specified as follows.

1. The gradient of c is computed in the $\pm\mathbf{x}$ and $\pm\mathbf{y}$ directions for each walker at state 1 and compared to a motion threshold m.
2. If any directional gradient is above m, the walker tries to move one lattice spacing in that direction.
3. It will not move onto a site already occupied, and will continue attempting to move either until it succeeds or until it progresses back to state 2.

Starting with walkers randomly distributed in space and a central pacemaker giving out signals periodically, simulations of the model show that the walkers move and aggregate to form highly ramified radial network, in agreement with experimental observations.

Another AW model is provided by Ben-Jacob *et al.* [1994], in which the active walkers perform random-walk motion within a flexible boundary on a triangular lattice. Equation (27) is replaced by

$$\frac{\partial c(\mathbf{r},t)}{\partial t} = D\nabla^2 c(\mathbf{r},t) - \sum_i \delta(\mathbf{r}-\mathbf{R}_i)\min[c_r, c(\mathbf{r},t)], \quad (28)$$

where $\mathbf{R}_i$ is the location of the ith walker. The last term implies that each walker consumes nutrients at a fixed rate c_r if sufficient food is available, and otherwise consumes the whole available amount.

Walker i has an internal energy w_i which evolves according to

$$\frac{dw_i}{dt} = \min[c_r, c(\mathbf{r},t)] - e \quad (29)$$

where e is a constant. For $w_i = 0$, the walker does not move; for $w_i > w_T$, a threshold, the walker divides into two. Compared to the previous model, the action of the walker here is simpler; also, the walker always decreases c, never adding to it. Reproduction is a new feature here, too.

When a segment of the boundary confining the walkers is hit by walkers N_c times, that segment is moved one lattice spacing forward while the boundary remains unbroken. (N_c represents the substrate's degree of hardness.) Thus the length of the boundary can only increase with time. Simulations of the model give ramified patterns, with the degree of ramification depending on the two parameters N_c and $P[\equiv c(\mathbf{r},0) = \text{const}]$. The pattern becomes denser as P increases or N_c decreases, in agreement with experiments.

4.3. *Intelligent swarms*

Much can be learned from ants as active walkers. When taken seriously and applied to social and business problems, many surprising and beneficial results are obtained [Bonabeau & Théraulaz, 2000].

Take the traveling salesman problem as an example, i.e. to find the shortest route to visit all N cities without visiting any city more than once. It is a computing heavy problem: For 15 cities, there are billions of route possibilities. An AW solution is to use a virtual ant to visit all cities, randomly at first. The path is laid down with virtual pheromone, such that the amount of the chemical is inversely proportional to the overall length of the tour. The process is repeated with a colony of ants. Since the pheromone evaporates (a negative feedback), links in short routes will eventually contain significant more pheromone than those in long routes. The pheromone concentration along the routes represents the landscape, and the ants, as active walkers, prefer to go through those links with high concentration of pheromone (a positive feedback). This AW approach, when combined with local search methods, out performs other heuristic methods in solving the problem, even though it may not always find the shortest route [Bonabeau *et al.*, 1999]. The same technique can be applied to other networking problems, like the optimization problem in cargo shipping business — for example, it did help Southwest Airlines to save money [Mucha, 2002].

4.4. *Artificial societies*

Active walk computer models were developed by Epstein and Axtell [1996] to show that social structures and collective behaviors — such as group formation, cultural transmission, combat and trade — can arise spontaneously from simple interaction of individuals. Unfortunately, no real data are included in their book. The landscape in their models is called "sugarscape," and the active walkers, "agents." ASCAPE is the platform to perform such modeling [Inchiosa & Parker, 2002], which can be downloaded from the Brookings Institution website.[4] Another platform is SWARM.[5] Both are free for noncommercial use.

[4] http://www.brook.edu/es/dynamics/models/ascape
[5] http://www.swarm.org

4.5. *Serial decision making processes*

Decision making is usually done serially. The possible choices (y) available at each decision step (n) may assume different fitness (z), where $n = 0, 1, 2, \ldots$ But once a decision is made at a certain step, the fitness of the options that come later could be affected and hence modified, so that $z = z(y(n))$. This process can be modeled by AW in a 3D discrete space, (n, y, z). For simplicity, we assume that only one option is actually picked at each decision step. Arrange the options at each step along the y direction, as a function of n; plot the corresponding fitness along z; an active walker moves on the (n, y) plane, with n increasing by one each time. As the walker moves forward, the fitness landscape is changed, affecting how the walker chooses its next step. The walker's location at step n in the 3D space is given by $(n, y(n), 0)$. In this model, the walker actually jumps from one (y, z) plane to the next (y, z) plane, and may have $y(n + 1) \neq y(n)$. Of course, if needed, the option y could be generalized to a multidimensional vector $\mathbf{y}$.

Active walk models like this are those used in the study of adaptive competition [Savit *et al.*, 1999] and evolution of financial market [Friedman, 2001]. Another example occurs in literature translation, suggested by Lu [1999]. Lu observes that there is more than one way to translate a certain sentence in a piece of literature such as *Jane Eyre*, from English to Chinese. However, the choice is narrowed and affected by the particular translation of the previous sentence. The process is like that in an AW.

4.6. *Other applications*

Active walk is employed by Lam *et al.* [1998] to model increasing returns in economics. Schweitzer [1998] uses AW to model the migration and economic agglomeration in a system of employed and unemployed workers, which respond to local wage gradients. The spatiotemporal distribution of workers is studied both analytically and by computer simulations.

A recent interesting application of AW is due to White and Harary [2004]. They construct graphs to study mathematically how active walkers, such as ants, can find the shortest paths in a maze by having only local knowledge of the maze, which they call "collective geodesics" property.

The applicability of active walks to educational reforms in the European Union is discussed by Pyla [2005].

5. Active Walks in History

History is the most important discipline of study [Lam, 2002]. Yet, the link between history and science is underdeveloped.

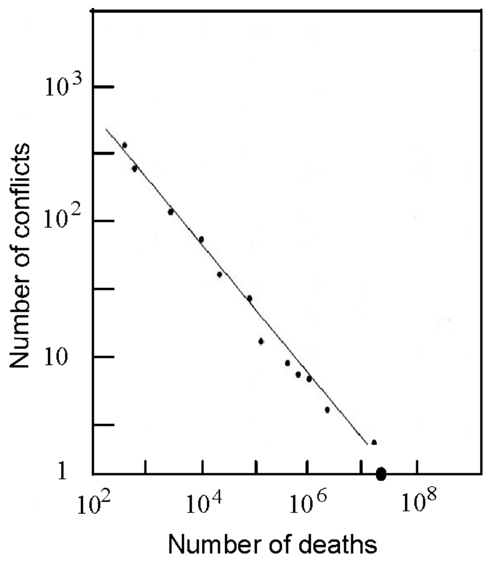
5.1. *What really is history?*

Science is the study of nature and a means to understand it in a unified way. Nature, of course, includes all material systems. The system investigated in history is a (biological) material system consisting of *Homo sapiens*. Consequently, history is a legitimate branch of science, like physics, biology, paleontology and so on. In other words, history is not a subject that is beyond the domain of science. History can be studied scientifically [Lam, 2002].

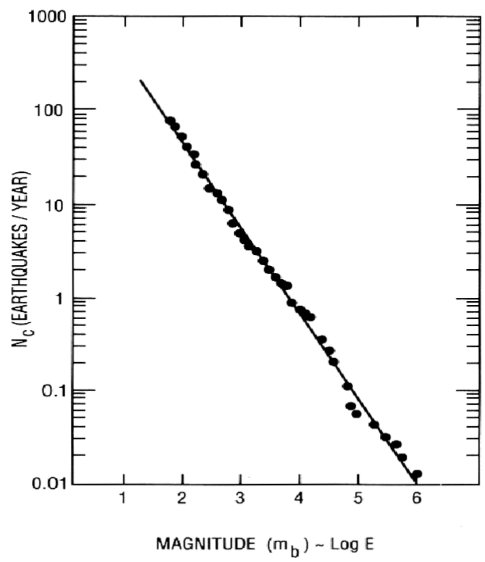
By definition, history is about past events and is irreproducible. In this regard, it is like the other "historical" sciences such as cosmology, astronomy, paleontology and archeology. The way historical sciences advance is by linking them to systems presently exist, which are amenable to tests. For example, in astronomy, the color spectra of light emitted in the past from the stars and received on earth can be compared with those observed in the laboratory; the identity of the elements existing in stars is then identified. Similarly, the psychology, thoughts and behaviors of historical players can be inferred from those of living human beings, which can be learned by observations, experimentations and neurophysiological probes [Feder, 2005].

The system under study in history is a many-body system. In this system, each "body" is a human being, called a "particle" here; these particles have internal states (due to thinking, memory, etc.) which sometimes can be ignored. Each constituent particle is a (nonquantum mechanical) classical object and is distinguishable, i.e. each particle in the system can be identified individually. This many-body system is a heterogeneous system, due to the different sizes, ages, races ... of the particles.

A historical process, expressed in the physics language, is the time development of a subset or the whole system of *Homo sapiens* that existed during a time period of interest in the past. History is therefore the study of the past dynamics of this system. *Historical processes are stochastic*, *resulting*



(a)



(b)

Fig. 19. (a) Statistical distribution of war intensities. Eighty-two wars from 1820 to 1929 are included; the dot on the horizontal axis comes from World War I. The graph is a log–log plot; a straight line indicates a power law. (b) Distribution of earthquake sizes in the New Madrid zone in the United States from 1974 to 1983 [Johnston & Nava, 1985]. The points show the number of earthquakes with magnitude larger than a given magnitude m.

from a combination of contingency and necessity. In modeling, contingency shows up as probability and necessity is represented by rules in the model. The situation is like that in a chess or soccer game. There are a few basic rules that the players have to obey, but because of contingency, the detail play-by-play of each game is different. In principle, someone with sufficient skills and patience can guess the rules governing historical processes, like those in a chess or soccer game.

In some cases, these two ingredients of contingency and necessity, through self-organization, may combine to give rise to discernable historical trends or laws. In other cases, either no laws exist at all or the laws are not recognized by whoever is studying them. Whether there actually exist historical laws cannot be settled by speculations or debates, no matter how good these speculations or debates are. *A historical law exists only when it is found and confirmed.* Furthermore, any historical law — like that in physics — has its own range of validity, which may cover only a limited domain of space and time.

Yet most people, including many historians, do not believe that any historical law could exist [Gardiner, 1959]. They are wrong. Figure 19(a) shows a historical law; it does exist. This power law on the statistics of war deaths is due to Richardson [1960]. Similar power laws are found in the distribution of earthquake intensities, called Gutenberg–Richter law [Fig. 19(b)], in the ranking of city populations, and in many other systems [Zipf, 1949]. The fact that human events like wars obey the same statistical law as inanimate systems indicates that the human system does belong to a larger class of dynamical systems in nature, beyond the control of human intentions and actions, individually or collectively. More historical laws are given below.

5.2. *Two quantitative laws and a quantitative prediction in Chinese history*

China has a long, unbroken history, which is probably the best documented [Huang, 1997]. The dynasties from Qin to Qing ranges from 221 B.C. to 1912, with 31 dynasties and 231 regimes spanning a total of 2133 years [Morby, 2002]. (A regime is the reign of one emperor; a dynasty may consist of several regimes.) Some of these dynasties overlap with each other in time.

Let τ_R be the regime lifetime, and τ_D the dynasty lifetime; both are integers measured in years. The histogram of τ_R is found to obey a power

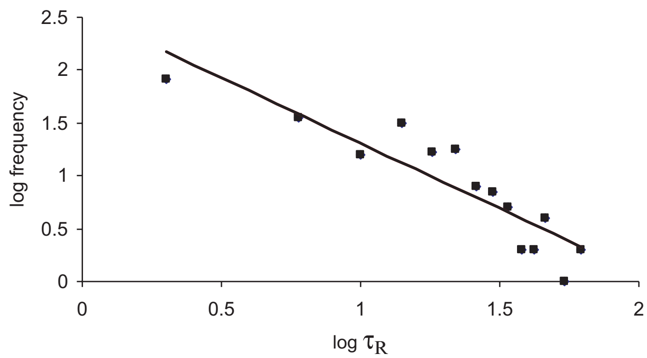


Fig. 20. Log–log plot of the histogram of regime lifetime τ_R, from 221 B.C. to 1912 in the Chinese history. The exponent of the straight-line fit is −1.3. (Bin width of the histogram is 4 years.)

law (Fig. 20), with an exponent equal to −1.3. This result implies that the dynamics governing regime changes is not completely up to the emperors, statistically speaking, but share some common traits with other complex systems such as those displayed in Fig. 19. This is the first quantitative law about Chinese history.

The second quantitative law is shown in Fig. 21, where the lifetimes of 26 dynasties are arranged in a monotonic decreasing order. Figure 21 is a Zipf plot, with the largest τ_D assigned rank 1, the second largest rank 2, etc. The same rank is assigned to separate τ_D that is identical to each other, to ensure that the curve is monotonic decreasing, i.e. no horizontal parts, and hence there are less than 31 data points in Fig. 21. They fall on two straight lines, which I name the *bilinear effect*. What it means is that (1) the "curse of history", as Chinese dynasties are concerned, does exist. (2) A dynasty can survive every 3.5 years if it lasts 59 years or less; beyond that, every 25.6 years — dynasty lifetime is discrete, or "quantized." (3) There is a transition point, at around $\tau_D = 59$ years, separating these two different behaviors.

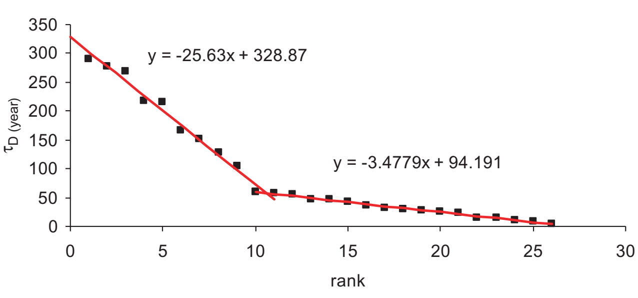


Fig. 21. The bilinear effect: Data points in a Zipf plot fall on two straight lines, with a transition point at the crossing. The data here are the lifetime of Chinese dynasties τ_D, from Qin to Qing. The transition occurs at around $\tau_D = 59$ years. The longest $\tau_D = 290$ years, the lifetime of Tang.

This is the phenomenon that a human entity, a dynasty in this case, becomes stronger after existing for a period of time. The mere fact of survival reinforces its strength, through adaptive learning, restructuring or other means. Similar behavior is known to exist in the lifetime of corporations or biological species. What is surprising here is the presence of two linear lines and a sharp transition point.

The bilinear effect, exemplified in a Zipf plot, is a general phenomenon found also in other human affairs and complex systems. It constitutes a *new* class of behavior in Zipf plots, apart from the two well-known classes of power laws [Zipf, 1949] and stretched exponents [Laherrère & Sornette, 1998].

Here is a *quantitative* prediction derived from Fig. 21: Any dynasty after Qing, if exists, either (1) would last 290 years or less, and fall more or less on the two lines in Fig. 21, or (2) would end definitely and exactly in the year 329.

Note that Fig. 21, in contrast to Fig. 20, is not a statistical plot, and this prediction is not a statistical prediction.[6] As far as I know, no other quantitative nonstatistical historical laws and predictions are known, to the historians or others. Note that these two laws and the prediction concerning Chinese history are model independent.

5.3. *Modeling history by active walks*

An important step towards the scientific study of any subject is to pick the right tool to tackle it. Historical processes are stochastic (i.e. with probability involved somewhere), resulting from necessity and contingency. The kind of physics suitable for handling many-body systems ingrained with contingency is statistical physics. Furthermore, the historical system is an *open* system with constant exchange of energy and materials with the environment and is never in equilibrium. Thus, for history, the appropriate tool is the stochastic methods developed in the statistical physics of nonequilibrium systems [Lam, 1998; Paul & Baschnagel,

[6]These two laws and the prediction in Chinese history were first presented by Lui Lam at the March meeting of APS, Montreal, 2004, and summarized in [Lam, 2006].

1999; Sornette, 2000]. In particular, AW can be used to model history. In fact, a common metaphor for history is that it is like a river flowing; people talk about the "river of history." This metaphor is not so off mark if the water flowing in the river is able to reshape the landscape as it flows and the river is allowed to branch from time to time under certain conditions. Active walk is a natural in matching such a metaphor. It is then no surprise that a whole class of probabilistic AW models are found to be relevant in studying history [Lam, 2002].

For example, (1) the two-site AW model (see Sec. 4 of Part I) is able to explain the real case in economic history that an inferior product such as the QWERTY keyboard [David, 1986] can actually win out in the market [Lam, 2002]. Other examples are the competition between Apple computers and PC's, as well as VHS and Beta videotapes.

(2) The active-walk aggregation (AWA) model is able to shed light on the debate in evolutionary history, initiated by Stephen Gould. The question raised by Gould [1989] is that if life's "tape" is replayed, will history repeat itself and humans still be found on earth? Gould's answer is "yes"; the AWA model says "maybe" [Lam, 1998]. It is "maybe" because if the world lies in the sensitive zone (see Fig. 6), then the growth outcome may not be repeatable; otherwise, it is repeatable.

(3) An intriguing prediction in social history was given by Fukuyama [1989], which asserts that every human being needs two kinds of satisfaction, namely, economic well-being and "recognition," with the latter meaning respect by others. Since only liberal democratic society can provide these two satisfactions, argued Fukuyama, all societies will end up as liberal democratic societies,

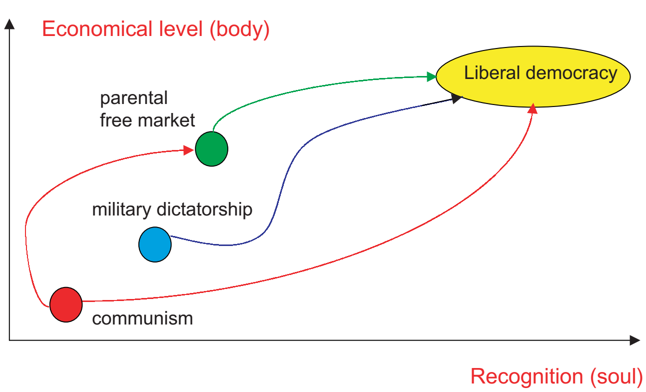


Fig. 22. Sketch of an active walk model for the evolution of political systems.

given enough time. And that will be the end of history, if history is understood to be directional change in societal forms. A phenomenological AW model with multiwalkers can be used to test this prediction [Lam, 2002]. In this model, each country is represented as an active walker, a particle, moving on a common deformable landscape in a 2D space. The problem will be to find out, under what conditions, if any, all the particles will cluster together at the location corresponding to a liberal democratic society (Fig. 22). It is thus a problem of clustering of active walkers on a 2D landscape. (See Sec. 7.4 for more details.)

5.4. *History in the future*

The importance of history can be seen, for example, through its negative impact on human lives. Powerful political leaders could mistake an unproven historical hypothesis as firm theory, apply it to a confined population and cause millions of death in a few short years [Lam, 2002]. Another example is the recent massive protests in China due to different interpretation of past history involving two countries (Fig. 23). Yet, in spite of its importance, the physical basis of history is unrecognized by most historians. For instance, in the historiography textbook *The New Nature of History* [Marwick, 2001] the alleged "fundamental" differences between history and the sciences are listed:

1. Fundamental difference in the subject of study: natural sciences concern natural and physical worlds; history concerns human beings and human society, very different in character.
2. No controlled experiments by historians.
3. Historians develop theories and theses, but are not concerned with developing laws and theories like that in sciences.
4. History studies do not have prediction power.
5. Relations and interactions in history studies are not expressed mathematically.
6. Historians report their findings in prose (articles or books), not in terse research articles.

Unfortunately, all six points are wrong, for the following reasons.

1. As explained in Sec. 5.1, human beings and thus human society are material systems, which are part of the natural sciences. Human society share same characteristics as other inanimate complex systems, as demonstrated in Figs. 19–21.

(a)

(b)

Fig. 23. Protests in China, April 2005. (a) "FACE HISTORY" is the slogan on the left placard. (b) "PROTECT DIAOYUDAO" is a historical issue also raised in the protests. Diaoyudao, or diaoyutai, is a group of tiny islands in the East China Sea. The "protect diaoyutai" movement was started by overseas Chinese students in the United States at the end of 1970 [The Seventies Monthly, 1971].

2. Some physical disciplines like astronomy and archeology also do not have controlled experiments.
3. It is untrue that all historians are not concerned with developing laws and theories in history. Some tried, not very successfully, partly due to their inadequate training in using scientific tools. Historical laws do exist, as shown in Sec. 5.2 and Figs. 19–21.
4. History studies, like that in Sec. 5.2, do have prediction power.
5. Relations and interactions in history studies can be expressed mathematically. An example is the landscape theory of Axelrod and Bennett [1993] to show how and why 17 European nations in Second World War aligned themselves into two large groups. The pairwise propensities between nations are assigned numerical values, and the configuration energy in the (fixed) landscape is given in equations.[7]
6. Historians do report their findings in research articles, terse or not. That is why there exist quite a number of history journals, such as *History and Theory* and *American Historical Review*. It is true that many historians still skip the peer-reviewed journals and directly report their findings in books — not a healthy thing for the history discipline, epistemologically speaking [Lam, 2002]. These are actually popular history books, like the popular science books written by physicists.[8] In the case of the history profession, there are at least three reasons behind this practice. (1) Many research results in history are still at the data gathering and empirical analysis stage, not very technical and can be presented in narratives. (2) There is enough number of readers out there who are willing to pay to find out what happened to their ancestors or their own kind in the past. In contrast, not that many will pay to learn what happened to the electrons. Bad for physics. (3) Historically, before history became a professional discipline in the universities in the second half of the nineteenth century, historians had to earn their living by writing books that are readable and saleable to the public [Stanford, 1998]. In other words, writing popular history books was a survival need for historians, a tradition carried over up to now.

These errors are due to misunderstanding of the nature of science, and the neglect of the material

[7]See [Galam, 1998] for a comment on this work, and the following response by the original authors.

[8]The unique characteristics of popular science books and how to integrate them into science teaching are discussed in [Lam, 2001, 2005b, 2005c].

basis of the historical system itself. The inadequate science training received by historians, past and present, explains why they failed to find historical laws. For example, the Chinese dynasty data have been in existence for 93 years; the plots in Figs. 20 and 21 could be carried out by hand without computers, and even by high school students. But unless one knows about power laws and the existence of Zipf plots, there is no motivation to do so. And these are current topics in the study of complex systems. Ironically, Zipf plots were first done by George Zipf (1902–1950), a Harvard linguist, with data from the humanities and social sciences.

Recently and surprisingly, while the importance of history is well recognized in Hong Kong, a history department is threatened with closure because it fails to attract enough number of students [Xie, 2005]. Anyway, it is time for all history departments to revamp their curriculum, by increasing the mathematical skills of students, going beyond story telling and making history research more technical and scientific, and creating a course on the physics of history (or histophysics [Lam, 2002, 2004]). This revamp will help current students to become better historians after they graduate, and may appeal to a new class of incoming students who have a technical background but feel attracted more to the humanities than the traditional sciences.

6. Physics, Social Science and Complex Systems

The conventional wisdom that "science consists of natural science and social science" is wrong. The correct answer is provided. Ten lessons from physics relevant to social science and complex systems research are given.

6.1. *What is science?*

Social science consists of anthropology, business and management, economics, education, environmental science, geography, government policy, law, psychology, social welfare, sociology and women's studies.[9] Social science is thus the study of the few-body or many-body problem of living *Homo sapiens* (excluding the medical aspects), while history is mostly about dead *Homo sapiens*. Obviously, the two are related, just as to understand annihilated electrons it helps to understand existing, moving electrons. Similar to the case of histophysics, physics can and should be used to study social systems, resulting in sociophysics [Montroll & Badger, 1974; Galam, 2004; Stauffer, 2003].[10]

As explained in Sec. 5.1, natural science is the study of all material systems. Anything — social science in particular — connected with *Homo sapiens*, a material system, is a legitimate part of natural science. Consequently, the usual understanding that

$$\text{science} = \text{natural science} + \text{social science} \tag{30}$$

is simply wrong. The correct expression should be

$$\begin{aligned}\text{science} &= \text{natural science}\\ &= \text{nonliving systems} + \text{living systems}\end{aligned} \tag{31}$$

whereas

$$\begin{aligned}\text{living systems} = {} & \text{nonhuman biological systems}\\ & + \text{human beings}\end{aligned} \tag{32}$$

Combining Eqs. (31) and (32), we have

$$\begin{aligned}\text{science} &= \text{natural science}\\ &= \text{physical science} + \text{social science}\\ &> \text{social science}\end{aligned} \tag{33}$$

where physical science includes not just physics, but biology, chemistry, etc.[11]

[9]See http://www.sosig.ac.uk/. Generally speaking, history together with philosophy, religions, languages, literature, art and music make up the humanities. On the other hand, while history is listed in the humanities at Stanford University, it is included in the social sciences at UC Berkeley and San Jose State University.

[10]A popular account of the physics of society is given by Ball [2004].

[11]How do humanities fit in? The aim of literature, music and art in the humanities is to stimulate the human brain — through arrangement of words or colors, sound or speech, or shape of things — to achieve pleasure and beauty, or their opposites, via the neurons and their connecting patterns [Pinker, 1997]. The brains, some sort of computer, of the creator and the receiver at the two ends of this process are heavily involved. The scientific development of these disciplines as complex systems is at a primitive level, and that is why they are separated from the social sciences, which are at an intermediate level. Linguistic is the study of the tools involved in written words and speeches, supporting the three disciplines mentioned above. History, by its nature, belongs more to the social sciences. With this understanding, Eq. (33) could be extended to read: science = natural science = physical science + social science + humanities. Or, equivalently, science = physical science + human science, where human science = social science + humanities. In short, everything under the sun and not just under the sun — everything in the universe — is a legitimate topic in science, unified at the fundamental level [Wilson, 1998].

6.2. *Ten lessons from physics*

Since a human being is the ultimate complex system, it is not hard to see that social systems are also complex systems. Complex systems, as a research field, cut across every discipline, and allow the participants to reach others far from their own background [Lam & Naroditsky, 1992; Cowan *et al.*, 1994]. Apart from the universal gravity that connects a mass to all other masses in the universe, complex system study is probably the next powerful thing that allows this to happen to the participants.

However, complex system scientists have different backgrounds, coming respectively from physics, chemistry, biology, geology, ecology, economics, psychology and computer science, to name a few. Perhaps because of this diversity in background and training, there are sometimes unavoidable misconceptions and confusion carried by some newcomers, and even some practitioners, in this field. Take, for example, the distinction between fractals, chaos and complex systems. These are three very different concepts, even though there are overlaps between them [Lam, 1998, 2000]. In particular, a complex system need not be a fractal or chaotic. A human body is a complex system; the whole body is not fractal or chaotic, even though some components inside the body are. For instance, the heartbeat in a normal person is chaotic; the bronchial tree is a fractal.

Also due to their diverse background, some of these scientists in complex systems may not be aware of the valuable lessons learned in physics — through hundreds, if not thousands, years of research, which are common knowledge among physicists, at least the good ones. Here are ten such lessons, written with graduate students in mind.

1. Physics succeeded not because the systems under study are simple, but because we make simplifications or approximations. Yes, electrons are simpler than humans. But an elephant is not simple. Physics succeeded because we are very daring in making simplifications. In Newton's second law of motion, $F = ma$, the mass m is that of a point particle, i.e. a particle without any size. There is no such particle in reality; it is a simplification. When we drop an elephant from a plane, we treat it like a point mass and use $F = ma$. The same goes to planets when we calculate their orbits.
2. "The simpler the better" (TSTB) is our motto.[12] We use toy models. A toy model is known to be unrealistic but may contain an ingredient that is essential to the phenomenon we want to understand. The Ising model, invented by a graduate student named Ising to explain phase transition in a ferromagnet, is extremely simple and contains only one parameter. It works, at least in 2D and 3D. And it continues to provide surprises for many decades. (Historically, when the Ising model was proposed, there were no real materials that matched it — those were discovered much later. Nature is kind to us, in this case. But that is beyond the point.) By working with many toy models and studying how they behave, we find out what the essential ingredient is. In contrast, social scientists tend to use complicated models from the beginning.
3. We work at steps, one step at a time. We always try to solve and understand a model in 1D, before we move to 2D or 3D. If the model contains two ingredients, we will keep one ingredient first, study it thoroughly, before we add in the second ingredient. This procedure is like doing experiments — you should tune only one parameter each time, otherwise you will not know which parameter causes the changes.
4. We tackle a problem at multiple levels. For example, the flow of water is studied at the empirical level by the Bernoulli's equation, at the phenomenological level by the Navier–Stokes equation, the microscopic level by Monte Carlo simulations, and finally, at the artificial level by lattice gas automata [Lam, 2002]. The phenomenological level is worth emphasizing since it is less used by social scientists, especially after they have access to powerful computers. Computers were invented in the forties in the last century. Computational physics emerged in the fifties and sixties with main frames, and flourished in the seventies and after personal computers became widely available. Before the forties, without the help of any computer, physics had already matured. This was due largely to the success of phenomenological theories, which do not require information about the components making up a system. For example, the Navier–Stokes equation is derived

[12] The TSTB principle is at the core of modern science, which is also called Occam's razor: "What can be done with fewer is done in vain with more."

without knowing that water is made up of molecules. (The AW models in Sec. 5.3 are phenomenological models.)

5. We understand that not all models have equal rights. We discard those that are not supported by reliable experiments, unless they can still provide insights. To argue that any computer model created by humans match reality somewhere — if not in our universe, in parallel universes — is not helpful. First, parallel universes may or may not exist. Second, even if they existed, research is about the wise use of our energy and time and the distribution of funding. We do not do a problem simply because it is doable.
6. We know that the reduction method (working from bottom up) is always valid, in principle. But we still often do physics at one level up, because of practical considerations. For instance, working from pairwise Coulomb interaction between electrons, we can calculate the properties of metals, but that does not prevent us to work with the elementary-excitation picture because it is efficient to do so and a clear physical understanding is gained.
7. "The whole is more than the sum" in nonlinear complex systems is a well-known fact in physics. If the sum means the properties of *isolated*, individual components in the system adding together, the whole could be larger than the sum when their mutual interactions are taken into account. The whole usually refers to emerging properties which are not foreseen from the sums, either because the mutual interactions are neglected, or, if they are included, we are not smart enough to foresee them. No holism is called for.
8. We adhere to data. No model or theory is established unless they are compared with and supported by reliable data. Many computer simulations in social science forsake this route, and end up as computer "games" [Bankes, 2002].
9. "Simplicity can lead to complexity" is well proven in physics; it is redundant to reestablish that in the study of social sciences or complex systems. For example, a collection of electron pairs could lead to superconductivity. In fact, almost everything interesting and important in the world, humans included, come from a simple hamiltonian of electrons and ions, with very simple Coulomb interactions [Laughlin & Pines, 2000].
10. In physics and elsewhere, experiments are done with real apparatuses. Calling the runs of a computer code written for a simulation model experiments does not make the simulation model more respectable or more real. It confuses students, if not the scientists themselves.

Incidentally, two important tools used to study stochastic systems were first employed in the social sciences, not physics. One is the application of random walks (see Sec. 2.2 of Part I); the other is probability. Probability was first introduced to understand gambling in the seventeenth century, and later as a foundation for the insurance business — a branch of economics or social sciences. The use of probability in physical sciences came much later, in the year 1859, in the forms of Maxwell's velocity distribution and Darwin's evolution theory [Lestienne, 1998]. Furthermore, sociology was originally called "social physics" by its founder, Comte (1798–1857) [Timasheff, 1957].

These examples illustrate the fact that all systems in nature, living and nonliving, are unified at a fundamental level — which was well recognized by the Greeks, about 2600 years ago. Plato and Aristotle may get the details wrong, but they pursued knowledge as a whole. In short, in the study of history, social systems and complex systems, let us be inspired by the Greeks, guided by physics, and equipped with statistical physics.

7. Open Problems

The major open problem in AW is to develop general methods to study theoretically the walk of a single active walker (like what is done in random walk [Weiss, 1994; Hughes, 1995]) and the collective behavior of multiwalkers, and the time evolution and structure of the corresponding landscape. Barring that, here are some doable open problems, presented as research projects.

7.1. *Balls rolling down a soft inclined plane*

The model: Start with a triangular lattice on a soft inclined plan. Allow a ball to roll down from a fixed lattice site at the top line. Then,

1. It can go to either the left or the right lattice site, one line below.

2. When it passes a lattice site below, it makes a dip of depth 1 at that site. Then it goes down another line below with the same rule #1, and reaches the bottom line far down eventually.
3. The probability for the ball to jump from site i to a site j below is given by P_{ij}, which is proportional to $\exp[\beta(V(i) - V(j))]$. Here, β is the inverse "temperature," with $0 \le \beta \le \infty$. (This is a BAW model.)
4. After the first ball reaches the bottom and removed, release a second ball from the same initial site at top, and so on, until N balls are released, one at a time.

Note that we are not talking real balls here; they are just massless point particles.

The question is, What is the spatial distribution of the N balls at the bottom line when N is large or approaching infinity? We have partial answers. We know the answer in the two extreme cases.

1. For $\beta = 0$, the model becomes a downward random walk model — the softness of the inclined plan has no effect. The distribution of the N balls is a Gaussian curve [Ambegaokar, 1996].
2. For $\beta = \infty$, it is the deterministic (DAW) model. Every ball will follow exactly the same path as the ball before it, and all the balls will end up at one bottom site. The distribution at the bottom line is thus a delta function (for $N \to \infty$), with the delta located at a bottom site. This site could be any of the sites at the bottom, picked by the balls with a Gaussian probability.

But what is the answer for $0 < \beta < \infty$? To solve it, write a computer program to find the answer first, with varying β. Then try to find the answer analytically.

The significance of this problem: The Gaussian curve is very fundamental, and is related to downward random walk. The model proposed here could be the simplest 2D AW model as AW is concerned, like the hydrogen atom problem in quantum mechanics. And no real AW model, with coupled equations and no approximation, has been solved yet. (The two-site AW model is solved by mapping it to a 1D site-dependent probabilistic walk, and then solved as a random walk problem.) This model can be compared with the results from a desktop experiment, not yet done by anybody.

7.2. *Fractal surfaces in a Boltzman active walk*

A single active walker, in a 1D BAW walk, changes an initially flat surface into a fractal surface. In this model the walker, at every step, decreases the surface on its two adjacent sites and increases the height at its own site, with the average height of the surface conserved. The landscaping function $W(r)$ is given by $W(0) = 2$, $W(\pm 1) = -1$, and $W = 0$ otherwise. Computer simulation results [Pochy *et al.*, 1993] show that the scaling properties of the surface thickness σ_T belong to a new class differing from those of other thermally activated models [Family & Vicsek, 1991]. For example, σ_T is independent of the system size L, but is a function of the "temperature" T. Specifically,

$$\sigma_T(L,t) \sim T^{\gamma} g\left(\frac{t}{T^{\gamma/\beta(T)}}\right) \tag{34}$$

where $\gamma = 0.48$; $g(x) = \text{const}$ for $x \gg 1$, and $g(x) = x^{\beta(T)}$ for $x \ll 1$. Here β is not "inverse temperature" but the exponent defined by $\sigma_T \sim T^{\beta}$, and t is the time. Equation (34) has the same form as that in most other models [Barabási & Stanley, 1995], except that on the right-hand side, T replaces L and β is not a constant but temperature dependent.

The challenge is to develop a theory to explain this scaling law. The master equation approach for surface evolution or other methods collected in [Family & Vicsek, 1991] should be consulted. After that, do the same for the LDL when $W(0)$ is varied across 2 [Lam, 1997].

7.3. *Microscopic simulation of surface-reaction filamentary patterns*

In the surface-reaction filamentary patterns induced by a sequence of dielectric breakdowns presented in Sec. 5 of Part I, the mechanism is very complicated and involves the thermohydrodynamics of the liquid in the cell and other ingredients. The AW modeling in Sec. 5.4 there is a phenomenological description. In principle, the AW models and the parameters used can be derived from one level down, at the molecular level.

My suggestion is to start from lattice gas automata [Chen *et al.*, 1991]; more specifically, from

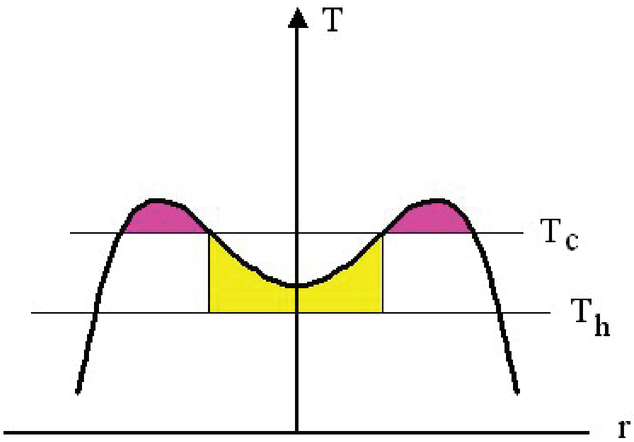


Fig. 24. Formation mechanism of a ring in surface-reaction filamentary patterns. Temperature T in the yellow region satisfies $T_h < T < T_c$, so that (invisible) dielectric breakdown first occurs there; the pink region appears later with $T > T_c$ where the (visible) ring is formed.

the lattice Boltzmann model [Shan & Chen, 1993] with heat sources added. The filament is where chemical reactions occur, which involve the chemical deposit on the two inner surfaces of the cell. Let us say that chemical reaction occurs at a threshold temperature T_c, and the dielectric breakdown at a threshold temperature T_h. These two processes release heat of amount Q_c and Q_h, respectively, as they occur. We do not know exactly the value of these temperatures. But the existence of a ring from which several filaments branch out (see Figs. 19 and 20 of Part I) indicates that $T_c > T_h$, as explained in Fig. 24.

Use a lattice Boltzmann model to describe the fluid flow in 2D. Assign a temperature T at the center so that $T_h < T < T_c$. Heat Q_h is added to this spot, causing the liquid around it to change into gas; the gas expands and pushes liquid away. Temperature at all lattice points is checked. At the location where $T > T_h$, heat Q_h is added. Similarly, at location with $T > T_c$, heat Q_c is added. The hydrodynamics of liquid and gas, and the transition from liquid to gas will be handled by the lattice gas automata. Numerical simulation should show where the chemical reactions occur, the formation and movement of bubbles around these locations, and the time-dependent heat and temperature distributions throughout the 2D space.

When the cell fluid is air, the case is simpler since the liquid–gas phase transition is absent. This problem could be handled first.

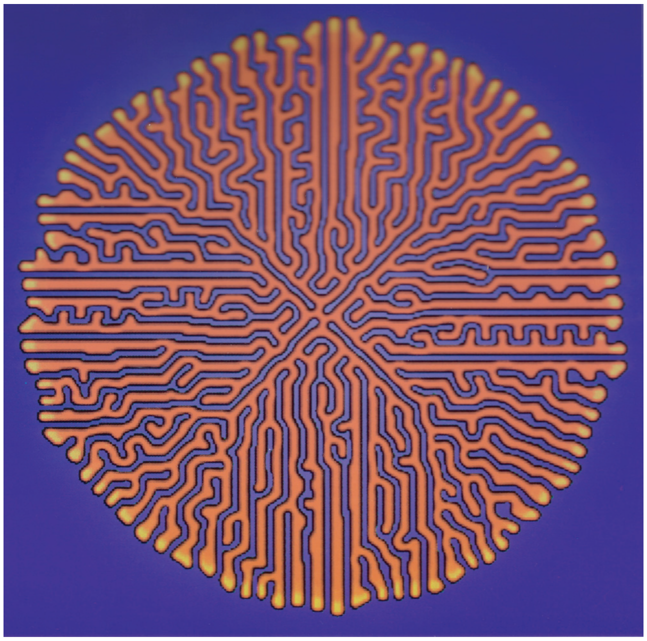
7.4. *Active walk modeling of the end of history problem*

In the AW model proposed in Sec. 5.3 to study the end of history problem, active walkers representing different countries are allowed to move in a 2D space. The y axis of this space is "economic satisfaction," which could be the "index of economic well-being" measured for different countries [Doyle, 2002a]. The x axis is "recognition," represented perhaps by the "happiness index" surveys taken from various countries [Doyle, 2002b]. These two indices for each country at different years give the trajectory of the corresponding particle in the 2D plane. At each point in the plane, a fitness potential is defined. The movement of each particle will change the fitness landscape and influences the movement of other particles. The stepping rule and landscaping rule of the AW have to be inferred from the existing trajectories. The future can then be predicted. A similar problem with simpler rules has been studied before in physics in another context, and the clustering of active walkers indeed occurs (see Sec. 2.1.2).

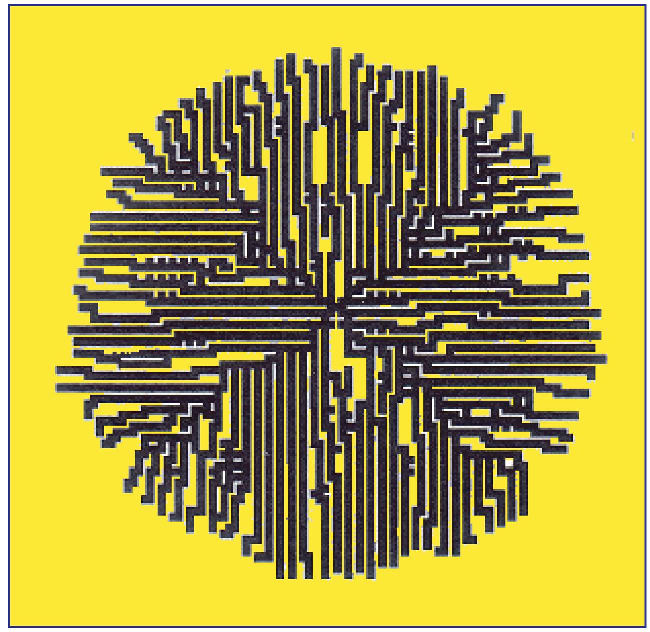
7.5. *Cellular neural network and active walk*

Cellular Neural Network (CNN) was proposed by Chua [1998] as a paradigm and tool to investigate complex systems, with broad applications in image and video signal processing, robotic and biological visions, higher brain functions, as well as pattern formations, autowaves, scroll waves and spatiotemporal chaos. Technically, CNN is a spatial arrangement of locally-coupled cells, where each cell is a dynamical system which has an input, an output, and internal states evolving according to some prescribed dynamical rules.

As such, CNN is a versatile and suitable platform to perform the calculations in AW. The internal states of a cell in a CNN could be the presence/absence of an active walker at the cell location, plus the velocity and any internal state of an active walker, as well as the height of the landscape function at that location. The input and output of a cell in a CNN come from the stepping rule, landscaping rule and the landscape's self-evolving rule — the three basic ingredients in an AW (see Sec. 3 of Part I), and the rules governing how the internal states of an active walker are changed in a specific application.



(a)



(b)

Fig. 25. Dense radial morphology, from active walk to cellular neural network. (a) A circular dendrite produced by CNN starting from a seed at the center (from book cover of [Chua, 1998]). (b) Similar dendrite from the PAW model, starting with four active walkers close to the center, with branching. (A similar dendrite is reported in [Lam *et al.*, 1992], where V_c should be 500 in the caption of Fig. 11.) Here, a square lattice of size 200×200 is used. The initial landscape is a cone with height $V_c = 500$ at the center, and zero when it intersects the boundary of the lattice. The landscaping function is W_1, with $W_0 = 5$, $r_1 = 12$, $r_2 = 15$; branching factor $\gamma = 0.5$. See [Lam, 1997] for definition of the parameters.

That CNN and AW can indeed achieve the same results is demonstrated in Fig. 25, showing the deep link between these two paradigms. The proposal is to use CNN to do the calculations called upon in various AW applications.

8. Discussions and Conclusion

Active walk has been applied successfully in modeling simple and complex systems. Whenever the system under study involves a sequence of choices and the choice at each step will affect the next choice, AW is applicable. (Decision making, management, career path, history and psychological processes are examples from the social sciences.) The landscape in the AW is the space of possible choices or states. If there is a metric (i.e. distance between the states is defined) in this state space, one can talk about neighborhoods of states, which is the case in most of the applications presented in this review. On the other hand, if there is no metric in the state space, one can still talk about connections between the states; this is the case of networks (see Sec. 4.3).

The landscape used in most applications of AW is a linear medium, in the sense that the landscaping action of the walker is linearly superposed to the existing landscape, or that the landscape could decay through diffusion described by a linear equation. However, it need not be so. The landscape could be a nonlinear medium, which is guaranteed to give rise to many surprising effects not yet explored.

Protein folding is very fast and is affected by the solution in which the protein is embedded [Creighton, 1972]. However, in the "conformational energy funnel" description [Frauenfelder *et al.*, 1991] of the folding process, a particle (representing the conformation assumed by the protein) is allowed to cascade down a *fixed* conformational energy landscape — the presence of solution is completely ignored. Yet it is precisely the solution that allows distant parts of the protein to communicate with each other. A more realistic model will have the particle replaced by an active walker; that is, the local energy landscape is modified in the presence of the particle — perhaps with local energy barriers lowered or disappearing — leading to a much faster process of downward cascading, and hence shorter time in protein folding. In other words, the funnel is now a deformable funnel, modified by the action of the particle.

Nature employs AW a lot. Apart from the many examples already mentioned, there is another one here. In general relativity, gravity results from the deformation of a space-time surface dragged along by the particle — an active walker.[13] It remains to be understood why Nature seems to prefer a potential theory, not just for elementary particles but even for complex systems, and whether some basic symmetry principles (like the gauge symmetry) are associated with the potential in these latter cases.

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